CHAPTER 7

Section 7.1

1.

a. $z_{\alpha/2} = 2.81$ implies that $\alpha/2 = 1 - \Phi(2.81) = .0025$, so $\alpha = .005$ and the confidence level is $100(1-\alpha)\% = 99.5\%$.

b. $z_{\alpha/2} = 1.44$ for $\alpha = 2[1 - \Phi(1.44)] = .15$, and $100(1-\alpha)\% = 85\%$.

c. 99.7% implies that $\alpha = .003$, $\alpha/2 = .0015$, and $z_{.0015} = 2.96$. (Look for cumulative area .9985 in the main body of table A.3, the Z table.)

d. 75% implies $\alpha = .25$, $\alpha/2 = .125$, and $z_{.125} = 1.15$.

2.

a. The sample mean is the center of the interval, so $\bar{x} = \dfrac{114.4 + 115.6}{2} = 115$.

b. The interval (114.4, 115.6) has the 90% confidence level. The higher confidence level will produce a wider interval.

3.

a. A 90% confidence interval will be narrower (See 2b, above) Also, the z critical value for a 90% confidence level is 1.645, smaller than the z of 1.96 for the 95% confidence level, thus producing a narrower interval.

b. Not a correct statement. Once and interval has been created from a sample, the mean μ is either enclosed by it, or not. The 95% confidence is in the general procedure, for repeated sampling.

c. Not a correct statement. The interval is an estimate for the population mean, not a boundary for population values.

d. Not a correct statement. In theory, if the process were repeated an infinite number of times, 95% of the intervals would contain the population mean μ.

4.

a. $58.3 \pm \frac{1.96(3)}{\sqrt{25}} = 58.3 \pm 1.18 = (57.1, 59.5)$

b. $58.3 \pm \frac{1.96(3)}{\sqrt{100}} = 58.3 \pm .59 = (57.7, 58.9)$

c. $58.3 \pm \frac{2.58(3)}{\sqrt{100}} = 58.3 \pm .77 = (57.5, 59.1)$

d. 82% confidence $\Rightarrow 1 - \alpha = .82 \Rightarrow \alpha = .18 \Rightarrow \alpha/2 = .09$, so $z_{\alpha/2} = z_{.09} = 1.34$ and the interval is $58.3 \pm \frac{1.34(3)}{\sqrt{100}} = (57.9, 58.7)$.

e. $n = \left[\frac{2(2.58)3}{1}\right]^2 = 239.62$ so n = 240.

5.

a. $4.85 \pm \frac{(1.96)(.75)}{\sqrt{20}} = 4.85 \pm .33 =$ (4.52, 5.18).

b. $z_{\alpha/2} = z_{.02/2} = z_{.01} = 2.33$, so the interval is $4.56 \pm \frac{(2.33)(.75)}{\sqrt{16}} =$ (4.12, 5.00).

c. $n = \left[\frac{2(1.96)(.75)}{.40}\right]^2 = 54.02$, so n = 55.

d. $n = \left[\frac{2(2.58)(.75)}{.2}\right]^2 = 93.61$, so n = 94.

6.

a. $8439 \pm \frac{(1.645)(100)}{\sqrt{25}} = 8439 \pm 32.9 =$ (8406.1, 8471.9).

b. $1 - \alpha = .92 \Rightarrow \alpha = .08 \Rightarrow \alpha/2 = .04$ so $z_{\alpha/2} = z_{.04} = 1.75$

7. If $L = 2z_{a/2}\frac{s}{\sqrt{n}}$ and we increase the sample size by a factor of 4, the new length is

$L' = 2z_{a/2}\frac{s}{\sqrt{4n}} = \left[2z_{a/2}\frac{s}{\sqrt{n}}\right]\left(\frac{1}{2}\right) = \frac{L}{2}$. Thus halving the length requires n to be increased fourfold. If $n' = 25n$, then $L' = \frac{L}{5}$, so the length is decreased by a factor of 5.

8.

a. With probability $1-a$, $z_{a_1} \le \left(\overline{X} - m\right)\left(\frac{s}{\sqrt{n}}\right) \le z_{a_2}$. These inequalities can be manipulated exactly as was done in the text to isolate m; the result is

$\overline{X} - z_{a_2}\frac{s}{\sqrt{n}} \le m \le \overline{X} + z_{a_1}\frac{s}{\sqrt{n}}$, so a $100(1-a)\%$ interval is

$\left(\overline{X} - z_{a_2}\frac{s}{\sqrt{n}}, \overline{X} + z_{a_1}\frac{s}{\sqrt{n}}\right)$

b. The usual 95% interval has length $3.92\frac{s}{\sqrt{n}}$, while this interval will have length

$\left(z_{a_1} + z_{a_2}\right)\frac{s}{\sqrt{n}}$. With $z_{a_1} = z_{.0125} = 2.24$ and $z_{a_2} = z_{.0375} = 1.78$, the length is

$(2.24 + 1.78)\frac{s}{\sqrt{n}} = 4.02\frac{s}{\sqrt{n}}$, which is longer.

9.

a. $\left(\overline{x} - 1.645\frac{s}{\sqrt{n}}, \infty\right)$. From 5**a**, $\overline{x} = 4.85$, $s = .75$, n = 20;

$4.85 - 1.645\frac{.75}{\sqrt{20}} = 4.5741$, so the interval is $(4.5741, \infty)$.

b. $\left(\overline{x} - z_a\frac{s}{\sqrt{n}}, \infty\right)$

c. $\left(-\infty, \overline{x} + z_a\frac{s}{\sqrt{n}}\right)$; From 4**a**, $\overline{x} = 58.3$, $s = 3.0$, n = 25;

$58.3 + 2.33\frac{3}{\sqrt{25}} = (-\infty, 59.70)$

10.

a. When n = 15, $2l\sum X_i$ has a chi-squared distribution with 30 d.f. From the 30 d.f. row of Table A.6, the critical values that capture lower and upper tail areas of .025 (and thus a central area of .95) are 16.791 and 46.979. An argument parallel to that given in Example 7.5 gives $\left(\frac{2\sum x_i}{46.979}, \frac{2\sum x_i}{16.791}\right)$ as a 95% C. I. for $m = \frac{1}{l}$. Since $\sum x_i = 63.2$ the interval is (2.69, 7.53).

b. A 99% confidence level requires using critical values that capture area .005 in each tail of the chi-squared curve with 30 d.f.; these are 13.787 and 53.672, which replace 16.791 and 46.979 in **a**.

c. $V(X) = \frac{1}{l^2}$ when X has an exponential distribution, so the standard deviation is $\frac{1}{l}$, the same as the mean. Thus the interval of **a** is also a 95% C.I. for the standard deviation of the lifetime distribution.

11. Y is a binomial r.v. with n = 1000 and p = .95, so E(Y) = np = 950, the expected number of intervals that capture m, and $s_Y = \sqrt{npq} = 6.892$. Using the normal approximation to the binomial distribution, $P(940 \le Y \le 960) = P(939.5 \le Y_{normal} \le 960.5) = P(-1.52 \le Z \le 1.52)$ $= .9357 - .0643 = .8714$.

Section 7.2

12. $$\bar{x} \pm 2.58\frac{s}{\sqrt{n}} = .81 \pm 2.58\frac{.34}{\sqrt{110}} = .81 \pm .08 = (.73, .89)$$

13.

a. $$\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}} = 1.028 \pm 1.96\frac{.163}{\sqrt{69}} = 1.028 \pm .038 = (.990, 1.066)$$

b. $$w = .05 = \frac{2(1.96)(.16)}{\sqrt{n}} \Rightarrow \sqrt{n} = \frac{2(1.96)(.16)}{.05} = 12.544 \Rightarrow n = (12.544)^2 \approx 158$$

14.

a. $89.10 \pm 1.96 \frac{3.73}{\sqrt{169}} = 89.10 \pm .56 = (88.54, .89.66)$. Yes, this is a very narrow interval. It appears quite precise.

b. $n = \left[\frac{(1.96)(.16)}{.5}\right]^2 = 245.86 \Rightarrow n = 246$.

15.

a. $z_a = .84$, and $\Phi(.84) = .7995 \approx .80$, so the confidence level is 80%.

b. $z_a = 2.05$, and $\Phi(2.05) = .9798 \approx .98$, so the confidence level is 98%.

c. $z_a = .67$, and $\Phi(.67) = .7486 \approx .75$, so the confidence level is 75%.

16. n = 46, $\bar{x} = 382.1$, s = 31.5; The 95% upper confidence bound =

$$\bar{x} + z_a \frac{s}{\sqrt{n}} = 382.1 + 1.645 \frac{31.5}{\sqrt{46}} = 382.1 + 7.64 = 389.74$$

17. $\bar{x} - z_{.01} \frac{s}{\sqrt{n}} = 135.39 - 2.33 \frac{4.59}{\sqrt{153}} = 135.39 - .865 = 134.53$ With a confidence level of 99%, the true average ultimate tensile strength is between (134.53, ∞).

18. 90% lower bound: $\bar{x} - z_{.10} \frac{s}{\sqrt{n}} = 4.25 - 1.28 \frac{1.30}{\sqrt{75}} = 4.06$

19. $\hat{p} = \frac{201}{356} = .5646$; We calculate a 95% confidence interval for the proportion of all dies that pass the probe:

$$\frac{.5646 + \frac{(1.96)^2}{2(356)} \pm 1.96\sqrt{\frac{(.5646)(.4354)}{356} + \frac{(1.96)^2}{4(356)^2}}}{1 + \frac{(1.96)^2}{356}} = \frac{.5700 \pm .0518}{1.01079} = (.513, .615)$$

20. Because the sample size is so large, the simpler formula (7.11) for the confidence interval for p is sufficient.

$$.15 \pm 2.58\sqrt{\frac{(.15)(.85)}{4722}} = .15 \pm .013 = (.137, .163)$$

21. $\hat{p} = \frac{133}{539} = .2468$; the 95% lower confidence bound is:

$$\frac{.2468 + \frac{(1.645)^2}{2(539)} - 1.645\sqrt{\frac{(.2468)(.7532)}{539} + \frac{(1.645)^2}{4(539)^2}}}{1 + \frac{(1.645)^2}{539}} = \frac{.2493 - .0307}{1.005} = .218$$

22. $\hat{p} = .072$; the 99% upper confidence bound is:

$$\frac{.072 + \frac{(2.33)^2}{2(487)} + 2.33\sqrt{\frac{(.072)(.928)}{487} + \frac{(2.33)^2}{4(487)^2}}}{1 + \frac{(2.33)^2}{487}} = \frac{.0776 + .0279}{1.0111} = .1043$$

23.

a. $\hat{p} = \frac{24}{37} = .6486$; The 99% confidence interval for p is

$$\frac{.6486 + \frac{(2.58)^2}{2(37)} \pm 2.58\sqrt{\frac{(.6486)(.3514)}{37} + \frac{(2.58)^2}{4(37)^2}}}{1 + \frac{(2.58)^2}{37}} = \frac{.7386 \pm .2216}{1.1799} = (.438, .814)$$

b. $$n = \frac{2(2.58)^2(.25) - (2.58)^2(.01) \pm \sqrt{4(2.58)^4(.25)(.25 - .01) + .01(2.58)^4}}{.01}$$

$$= \frac{3.261636 \pm 3.3282}{.01} \approx 659$$

24. n = 56, $\bar{x} = 8.17$, s = 1.42; For a 95% C.I., $z_{\alpha/2} = 1.96$. The interval is

$8.17 \pm 1.96\left(\frac{1.42}{\sqrt{56}}\right) = (7.798, 8.542)$. We make no assumptions about the distribution if percentage elongation.

25.

a. $n = \dfrac{2(1.96)^2(.25) - (1.96)^2(.01) \pm \sqrt{4(1.96)^4(.25)(.25 - .01) + .01(1.96)^4}}{.01} \approx 381$

b. $n = \dfrac{2(1.96)^2\left(\frac{1}{3}\cdot\frac{2}{3}\right) - (1.96)^2(.01) \pm \sqrt{4(1.96)^4\left(\frac{1}{3}\cdot\frac{2}{3}\right)\left(\frac{1}{3}\cdot\frac{2}{3} - .01\right) + .01(1.96)^4}}{.01} \approx 339$

26. With $q = l$, $\hat{q} = \overline{X}$ and $s_{\hat{q}} = \sqrt{\frac{l}{n}}$ so $\hat{s}_{\hat{q}} = \sqrt{\frac{\overline{X}}{n}}$. The large sample C.I. is then $\overline{x} \pm z_{a/2}\sqrt{\frac{\overline{x}}{n}}$. We calculate $\sum x_i = 203$, so $\overline{x} = 4.06$, and a 95% interval for l is $4.06 \pm 1.96\sqrt{\frac{4.06}{50}} = 4.06 \pm .56 = (3.50, 4.62)$

27. Note that the midpoint of the new interval is $\dfrac{x + \frac{z^2}{2}}{n + z^2}$, which is roughly $\dfrac{x+2}{n+4}$ with a confidence level of 95% and approximating $1.96 \approx 2$. The variance of this quantity is $\dfrac{np(1-p)}{(n+z^2)^2}$, or roughly $\dfrac{p(1-p)}{n+4}$. Now replacing p with $\dfrac{x+2}{n+4}$, we have $\left(\dfrac{x+2}{n+4}\right) \pm z_{\alpha/2}\sqrt{\dfrac{\left(\frac{x+2}{n+4}\right)\left(1 - \frac{x+2}{n+4}\right)}{n+4}}$; For clarity, let $x^* = x + 2$ and $n^* = n + 4$, then $\hat{p}^* = \dfrac{x^*}{n^*}$ and the formula reduces to $\hat{p}^* \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}^*\hat{q}^*}{n^*}}$, the desired conclusion. For further discussion, see the Agresti article.

Section 7.3

28.

a. 1.341

b. 1.753

c. 1.708

d. 1.684

e. 2.704

29.

a. $t_{.025,10} = 2.228$

b. $t_{.025,20} = 2.086$

c. $t_{.005,20} = 2.845$

d. $t_{.005,50} = 2.678$

e. $t_{.01,25} = 2.485$

f. $-t_{.025,5} = -2.571$

30.

a. $t_{.025,10} = 2.228$

b. $t_{.025,15} = 2.131$

c. $t_{.005,15} = 2.947$

d. $t_{.005,4} = 4.604$

e. $t_{.01,24} = 2.492$

f. $t_{.005,37} \approx 2.712$

31.

a. $t_{05,10} = 1.812$

b. $t_{.05,15} = 1.753$

c. $t_{.01,15} = 2.602$

d. $t_{.01,4} = 3.747$

e. $\approx t_{.025,24} = 2.064$

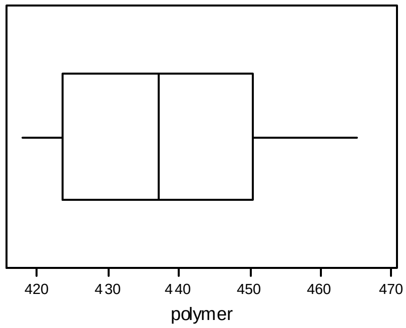
f. $t_{.01,37} \approx 2.429$

32. d.f. = n – 1 = 7, so the critical value for a 95% C.I. is $t_{.025,7} = 2.365$. The interval is

$$30.2 \pm (2.365)\left(\frac{3.1}{\sqrt{8}}\right) = 30.2 \pm 2.6 = (27.6, 32.8).$$

33.

a. The boxplot indicates a very slight positive skew, with no outliers. The data appears to center near 438.



b. Based on a normal probability plot, it is reasonable to assume the sample observations came from a normal distribution.

c. With d.f. = n – 1 = 16, the critical value for a 95% C.I. is $t_{.025,16} = 2.120$, and the interval is $438.29 \pm (2.120)\left(\frac{15.14}{\sqrt{17}}\right) = 438.29 \pm 7.785 = (430.51, 446.08)$. Since 440 is within the interval, 440 is a plausible value for the true mean. 450, however, is not, since it lies outside the interval.

34. n = 14, $\bar{x} = 8.48$, s = .79; $t_{.05,13} = 1.771$

a. A 95% lower confidence bound: $8.48 - 1.771\left(\frac{.79}{\sqrt{14}}\right) = 8.48 - .37 = 8.11$. With 95% confidence, the value of the true mean proportional limit stress of all such joints lies in the interval $(8.11, \infty)$. If this interval is calculated for sample after sample, in the long run 95% of these intervals will include the true mean proportional limit stress of all such joints. We must assume that the sample observations were taken from a normally distributed population.

b. A 95% lower prediction bound: $8.48 - 1.771(.79)\sqrt{1 + \frac{1}{14}} = 8.48 - 1.45 = 7.03$. If this bound is calculated for sample after sample, in the long run 95% of these bounds will provide a lower bound for the corresponding future values of the proportional limit stress of a single joint of this type.

35. n = 5, $\bar{x} = 2887.6$, s = .84.0; $t_{.025,4} = 2.776$

a. A 95% C.I. for the mean: $2887.6 \pm (2.776)\left(\frac{84}{\sqrt{5}}\right) \Rightarrow (2783.3, 2991.9)$

b. A 95% Prediction Interval: $2887.6 \pm 2.776(84)\sqrt{1+\frac{1}{5}} \Rightarrow (2632.1, 3143.1)$. The P.I. is considerably larger than the C.I., about 2.5 times larger.

36. n = 26, $\bar{x} = 370.69$, s = 24.36; $t_{.05,25} = 1.708$

a. A 95% upper confidence bound:

$$370.69 + (1.708)\left(\frac{24.36}{\sqrt{26}}\right) = 370.69 + 8.16 = 378.85$$

b. A 95% upper prediction bound:

$$370.69 + 1.708(24.36)\sqrt{1+\frac{1}{26}} = 370.69 + 42.45 = 413.14$$

c. Following a similar argument as that on p. 300 of the text, we need to find the variance of $\overline{X} - \overline{X}_{new}$: $V\left(\overline{X} - \overline{X}_{new}\right) = V\left(\overline{X}\right) + V\left(\overline{X}_{new}\right) = V\left(\overline{X}\right) + V\left(\frac{1}{2}\left(X_{27} + X_{28}\right)\right)$
$= V\left(\overline{X}\right) + V\left(\frac{1}{2}X_{27}\right) + V\left(\frac{1}{2}X_{28}\right) = V\left(\overline{X}\right) + \frac{1}{4}V\left(X_{27}\right) + \frac{1}{4}V\left(X_{28}\right)$
$= \frac{s^2}{n} + \frac{1}{4}s^2 + \frac{1}{4}s^2 = s^2\left(\frac{1}{2} + \frac{1}{n}\right)$. We eventually arrive at $T = \frac{\overline{X} - \overline{X}_{new}}{s\sqrt{\frac{1}{2}+\frac{1}{n}}} \sim$ t distribution with n – 1 d.f., so the new prediction interval is $\bar{x} \pm t_{\alpha/2,n-1} \cdot s\sqrt{\frac{1}{2}+\frac{1}{n}}$. For this situation, we have

$$370.69 \pm 1.708(24.36)\sqrt{\frac{1}{2}+\frac{1}{26}} = 370.69 \pm 30.53 = (39.47, 400.53)$$

37.

a. A 95% C.I. : $.9255 \pm 2.093(.0181) = .9255 \pm .0379 \Rightarrow (.8876, .9634)$

b. A 95% P.I. : $.9255 \pm 2.093(.0809)\sqrt{1+\frac{1}{20}} = .9255 \pm .1735 \Rightarrow (.7520, 1.0990)$

c. A tolerance interval is requested, with k = 99, confidence level 95%, and n = 20. The tolerance critical value, from Table A.6, is 3.615. The interval is
$.9255 \pm 3.615(.0809) \Rightarrow (.6330, 1.2180)$.

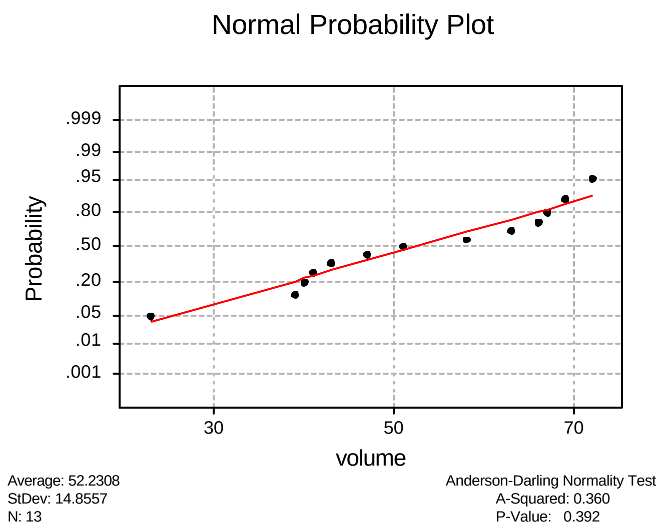
38. N = 25, $\overline{x} = .0635$, s = .0065

a. 95% P.I. : $.0635 \pm 2.064(.0065)\sqrt{1+\frac{1}{25}} = .0635 \pm .0137 \Rightarrow (.0498,.0772)$.

b. 99% Tolerance Interval, with k = 95, critical value 2.972 (table A.6):
$.0635 \pm 2.972(.0065) \Rightarrow (.0442,.0828)$.

39.

a.



Based on the above plot, generated by Minitab, it is plausible that the population distribution is normal.

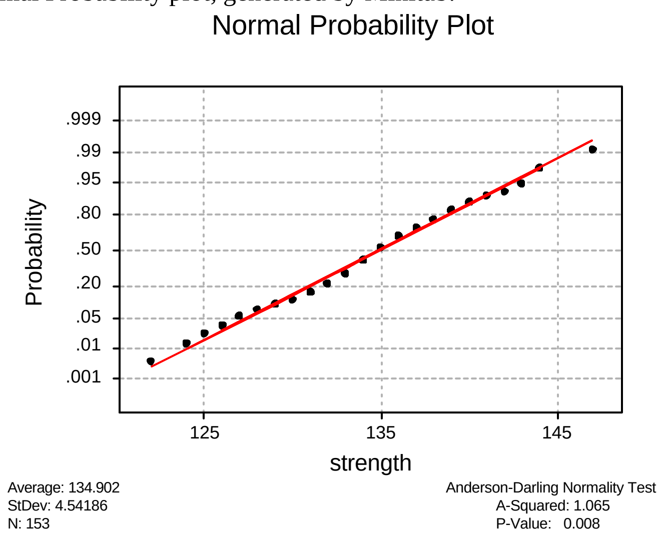
b. We require a tolerance interval. (from table A6, with 95% confidence, k = 95, and n=13, the tcv = 3.081.
$\overline{x} \pm (tcv)s = 52.231 \pm 3.081(14.856) = 52.231 \pm 45.771 \Rightarrow (6.460, 98.002)$

c. A prediction interval, with $t_{.025,12} = 2.179$:
$52.231 \pm 2.179(14.856)\sqrt{1+\frac{1}{13}} = 52.231 \pm 33.593 \Rightarrow (18.638, 85.824)$

40.

a. We need to assume the samples came from a normally distributed population.

b. A Normal Probability plot, generated by Minitab:



The very small p-value indicates that the population distribution from which this data was taken is most likely not normal.

c. 95% lower prediction bound:

$$52.231 \pm 2.179(14.856)\sqrt{1+\tfrac{1}{13}} = 52.231 \pm 33.593 \Rightarrow (18.638, 85.824)$$

41. The 20 d.f. row of Table A.5 shows that 1.725 captures upper tail area .05 and 1.325 captures uppertail area .10 The confidence level for each interval is 100(central area)%. For the first interval, central area = 1 – sum of tail areas = 1 – (.25 + .05) = .70, and for the second and third intervals the central areas are 1 – (.20 + .10) = .70 and 1 – (.15 + .15) = 70. Thus each interval has confidence level 70%. The width of the first interval is

$\dfrac{s(.687+1.725)}{\sqrt{n}} = \dfrac{.2412s}{\sqrt{n}}$, whereas the widths of the second and third intervals are 2.185 and 2.128 respectively. The third interval, with symmetrically placed critical values, is the shortest, so it should be used. This will always be true for a t interval.

Section 7.2

42.

a. $c^2_{.1,15} = 22.307$ (.1 column, 15 d.f. row)

b. $c^2_{.1,25} = 34.381$

c. $c^2_{.01,25} = 44.313$

d. $c^2_{.005,25} = 46.925$

e. $c^2_{.99,25} = 11.523$ (from .99 column, 25 d.f. row)

f. $c^2_{.995,25} = 10.519$

43.

a. $c^2_{.05,10} = 18.307$

b. $c^2_{.95,10} = 3.940$

c. Since $10.987 = c^2_{.975,22}$ and $36.78 = c^2_{.025,22}$, $P\left(c^2_{.975,22} \le c^2 \le c^2_{.025,22}\right) = .95$.

d. Since $14.61 = c^2_{.95,25}$ and $37.65 = c^2_{.05,25}$, $P\left(c^2_{.95,25} \le c^2 \le c^2_{.05,25}\right) = .90$.

44. n – 1 = 8, $c^2_{.025,8} = 17.543$, $c^2_{.975,8} = 2.180$, so the 95% interval for s^2 is $\left(\frac{8(7.90)}{17.543}, \frac{8(7.90)}{2.180}\right) = (3.60, 28.98)$. The 95% interval for s is $\left(\sqrt{3.60}, \sqrt{28.98}\right) = (1.90, 5.38)$.

45. n = 22 implies that d.f. = n – 1 = 21, so the .995 and .005 columns of Table A.7 give the necessary chi-squared critical values as 8.033 and 41.399. $\Sigma x_i = 1701.3$ and $\Sigma x_i^2 = 132{,}097.35$, so $s^2 = 25.368$. The interval for s^2 is $\left(\frac{21(25.368)}{41.399}, \frac{21(25.368)}{8.033}\right) = (12.868, 66.317)$ and that for s is $(3.6, 8.1)$ Validity of this interval requires that fracture toughness be (at least approximately) normally distributed.

46.

a. Using a normal probability plot, we ascertain that it is plausible that this sample was taken from a normal population distribution.

b. With s = 1.579 , n = 15, and $c^2_{.05,14} = 23.685$ the 95% upper confidence bound for s is $\sqrt{\frac{14(1.579)^2}{23.685}} = 1.214$

Supplementary Exercises

47.

a. n = 48, $\bar{x} = 8.079$, s^2 = 23.7017, and s = 4.868.
A 95% C.I. for m = the true average strength is

$$\bar{x} \pm 1.96\frac{s}{\sqrt{n}} = 8.079 \pm 1.96\frac{4.868}{\sqrt{48}} = 8.079 \pm 1.377 = (6.702, 9.456)$$

b. $\hat{p} = \frac{13}{48} = .2708$. A 95% C.I. is

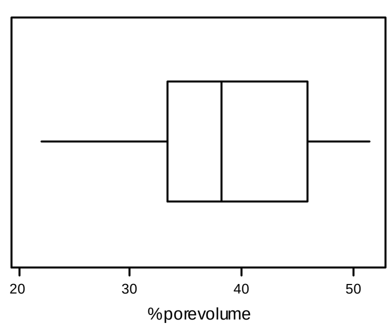
$$\frac{.2708 + \frac{1.96^2}{2(48)} \pm 1.96\sqrt{\frac{(.2708)(.7292)}{48} + \frac{1.96^2}{4(48)^2}}}{1 + \frac{1.96^2}{48}} = \frac{.3108 \pm .1319}{1.0800} = (.166, .410)$$

48. A 98% t C.I. requires $t_{a/2,n-1} = t_{.01,8} = 2.896$. The interval is

$$188.0 \pm 2.896\frac{7.2}{\sqrt{9}} = 188.0 \pm 7.0 = (181.0, 195.0).$$

49.

a. There appears to be a slight positive skew in the middle half of the sample, but the lower whisker is much longer than the upper whisker. The extent of variability is rather substantial, although there are no outliers.



b. The pattern of points in a normal probability plot is reasonably linear, so, yes, normality is plausible.

c. n = 18, $\bar{x} = 38.66$, s = 8.473, and $t_{.01,17} = 2.586$. The 98% confidence interval is

$$38.66 \pm 2.586\frac{8.473}{\sqrt{18}} = 38.66 \pm 5.13 = (33.53, 43.79).$$

50. $\bar{x} =$ the middle of the interval $= \dfrac{229.764 + 233.502}{2} = 231.633.$ To find *s* we use $width = 2\left(t_{.025,4}\right)\left(\dfrac{s}{\sqrt{n}}\right)$, and solve for *s*. Here, n = 5, $t_{.025,4} = 2.776$, and width = upper limit – lower limit = 3.738. $3.738 = 2(2776)\dfrac{s}{\sqrt{5}} \Rightarrow s = \dfrac{\sqrt{5}(3.738)}{2(2.776)} = 1.5055$. So for a 99% C.I., $t_{.005,4} = 4.604$, and the interval is

$$231.633 \pm 4.604\frac{1.5055}{\sqrt{5}} = 213.633 \pm 3.100 = (228.533, 234.733).$$

51.

a. $\hat{p} = \dfrac{136}{200} = .680 \Rightarrow$ a 90% C.I. is

$$\frac{.680 + \frac{1.645^2}{2(200)} \pm 1.645\sqrt{\frac{(.680)(.320)}{200} + \frac{1.645^2}{4(200)^2}}}{1 + \frac{1.645^2}{200}} = \frac{.6868 \pm .0547}{1.01353} = (.624, .732)$$

b. $$n = \frac{2(1.645)^2(.25) - (1.645)^2(.05)^2 \pm \sqrt{4(1.645)^4(.25)(.25 - .0025) + .05^2(1.645)^4}}{.0025}$$

$$= \frac{1.3462 \pm 1.3530}{.0025} = 1079.7 \Rightarrow \text{ use n = 1080}$$

c. No, it gives a 95% upper bound.

52.

a. Assuming normality, $t_{.05,15} = 1.753$, do s 95% C.I. for m is

$$.214 \pm 1.753\frac{.036}{\sqrt{16}} = .214 \pm .016 = (.198, .230)$$

b. A 90% upper bound for s, with $c^2_{.10,15} = 1.341$, is $\sqrt{\dfrac{15(.036)^2}{1.341}} = \sqrt{.0145} = .120$

c. A 95% prediction interval, with $t_{.025,15} = 2.131$, is

$$.214 \pm 2.131(.036)\sqrt{1 + \tfrac{1}{16}} = .214 \pm .0791 = (.1349, .2931).$$

53. With $\hat{q} = \frac{1}{3}(\overline{X}_1 + \overline{X}_2 + \overline{X}_3) - \overline{X}_4$, $s_{\hat{q}}^2 = \frac{1}{9}Var(\overline{X}_1 + \overline{X}_2 + \overline{X}_3) + Var(\overline{X}_4) =$ $\frac{1}{9}\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} + \frac{s_3^2}{n_3}\right) + \frac{s_4^2}{n_4}$; $\hat{s}_{\hat{q}}$ is obtained by replacing each $\hat{s}_i^2$ by s_i^2 and taking the square root. The large-sample interval for q is then $\frac{1}{3}(\overline{x}_1 + \overline{x}_2 + \overline{x}_3) - \overline{x}_4 \pm z_{a/2}\sqrt{\frac{1}{9}\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} + \frac{s_3^2}{n_3}\right) + \frac{s_4^2}{n_4}}$. For the given data, $\hat{q} = -.50$, $\hat{s}_{\hat{q}} = .1718$, so the interval is $-.50 \pm 1.96(.1718) = (-.84, -.16)$.

54. $\hat{p} = \frac{11}{55} = .2 \Rightarrow$ a 90% C.I. is

$$\frac{.2 + \frac{1.645^2}{2(55)} \pm 1.645\sqrt{\frac{(.2)(.8)}{55} + \frac{1.645^2}{4(55)^2}}}{1 + \frac{1.645^2}{55}} = \frac{.2246 \pm .0887}{1.0492} = (.1295, .2986).$$

55. The specified condition is that the interval be length .2, so $n = \left[\frac{2(1.96)(.8)}{.2}\right]^2 = 245.86$, so n = 246 should be used.

56.

a. A normal probability plot lends support to the assumption that pulmonary compliance is normally distributed. Note also that the lower and upper fourths are 192.3 and 228,1, so the fourth spread is 35.8, and the sample contains no outliers.

b. $t_{.025,15} = 2.131$, so the C.I. is

$$209.75 \pm 2.131\frac{24.156}{\sqrt{16}} = 209.75 \pm 12.87 = (196.88, 222.62).$$

c. K = 95, n = 16, and the tolerance critical value is 2.903, so the 95% tolerance interval is $209.75 \pm 2.903(24.156) = 209.75 \pm 70.125 = (139.625, 279.875)$.

57. Proceeding as in Example 7.5 with T_r replacing ΣX_i, the C.I. for $\frac{1}{l}$ is $\left(\frac{2t_r}{c^2_{1-a/2,2r}}, \frac{2t_r}{c^2_{a/2,2r}}\right)$ where $t_r = y_1 + ... + y_r + (n-r)y_r$. In Example 6.7, n = 20, r = 10, and t_r = 1115. With d.f. = 20, the necessary critical values are 9.591 and 34.170, giving the interval (65.3, 232.5). This is obviously an extremely wide interval. The censored experiment provides less information about $\frac{1}{l}$ than would an uncensored experiment with n = 20.

58.

a. $P(\min(X_i) \le \tilde{m} \le \max(X_i)) = 1 - P(\tilde{m}, < \min(X_i) or \max(X_i) < \tilde{m})$
$= 1 - P(\tilde{m}, < \min(X_i)) - P(\max(X_i) < \tilde{m})$
$= 1 - P(\tilde{m} < X_1, ..., \tilde{m} < X_n) - P(X_1 < \tilde{m}, ..., X_n < \tilde{m})$
$= 1 - (.5)^n - (.5)^n = 1 - 2(.5)^{n-1}$, from which the confidence interval follows.

b. Since $\min(x_i) = 1.44$ and $\max(x_i) = 3.54,$ the C.I. is (1.44, 3.54).

c. $P(X_{(2)} \le \tilde{m} \le X_{(n-1)}) = 1 - P(\tilde{m}, < X_{(2)}) - P(X_{(n-1)} < \tilde{m})$
= 1 – P(at most one X_I is below $\tilde{m}$) – P(at most one X_I exceeds $\tilde{m}$)
$1 - (.5)^n - \binom{n}{1}(.5)^1(.5)^{n-1} - (.5)^n - \binom{n}{1}(.5)^{n-1}(.5).$
$= 1 - 2(n+1)(.5)^n = 1 - (n+1)(.5)^{n-1}$
Thus the confidence coefficient is $1 - (n+1)(.5)^{n-1}$, or in another way, a $100(1 - (n+1)(.5)^{n-1})\%$ confidence interval.

59.

a. $\int_{(a/2)^{1/n}}^{(1-a/2)^{1/n}} nu^{n-1}du = u^n \Big]_{(a/2)^{1/n}}^{(1-a/2)^{1/n}} = 1 - \frac{a}{2} - \frac{a}{2} = 1 - a$. From the probability statement, $\frac{(a/2)^{1/n}}{\max(X_i)} \le \frac{1}{q} \le \frac{(1-a/2)^{1/n}}{\max(X_i)}$ with probability $1 - a$, so taking the reciprocal of each endpoint and interchanging gives the C.I. $\left(\frac{\max(X_i)}{(1-a/2)^{1/n}}, \frac{\max(X_i)}{(a/2)^{1/n}}\right)$ for q.

b. $a^{1/n} \le \frac{\max(X_i)}{q} \le 1$ with probability $1 - a$, so $1 \le \frac{q}{\max(X_i)} \le \frac{1}{a^{1/n}}$ with probability $1 - a$, which yields the interval $\left(\max(X_i), \frac{\max(X_i)}{a^{1/n}}\right)$.

c. It is easily verified that the interval of **b** is shorter – draw a graph of $f_U(u)$ and verify that the shortest interval which captures area $1 - a$ under the curve is the rightmost such interval, which leads to the C.I. of **b**. With $a = .05,$ n = 5, $\max(x_I)$=4.2; this yields (4.2, 7.65).

60. The length of the interval is $\left(z_g + z_{a-g}\right)\dfrac{s}{\sqrt{n}}$, which is minimized when $z_g + z_{a-g}$ is minimized, i.e. when $\Phi^{-1}(1-g) + \Phi^{-1}(1-a+g)$ is minimized. Taking $\dfrac{d}{dg}$ and equating to 0 yields $\dfrac{1}{\Phi(1-g)} = \dfrac{1}{\Phi(1-a+g)}$ where $\Phi(\bullet)$ is the standard normal p.d.f., whence $g = \dfrac{a}{2}$.

61. $\tilde{x} = 76.2$, the lower and upper fourths are 73.5 and 79.7, respectively, and $f_s = 6.2$. The robust interval is $76.2 \pm (1.93)\left(\dfrac{6.2}{\sqrt{22}}\right) = 76.2 \pm 2.6 = (73.6, 78.8)$.

$\bar{x} = 77.33$, s = 5.037, and $t_{.025,21} = 2.080$, so the t interval is

$77.33 \pm (2.080)\left(\dfrac{5.037}{\sqrt{22}}\right) = 77.33 \pm 2.23 = (75.1, 79.6)$. The t interval is centered at $\bar{x}$, which is pulled out to the right of $\tilde{x}$ by the single mild outlier 93.7; the interval widths are comparable.

62.

a. Since $2l\Sigma X_i$ has a chi-squared distribution with 2n d.f. and the area under this chi-squared curve to the right of $c^2_{.95,2n}$ is .95, $P\left(c^2_{.95,2n} < 2l\Sigma X_i\right) = .95$. This implies that $\dfrac{c^2_{.95,2n}}{2\Sigma X_i}$ is a lower confidence bound for l with confidence coefficient 95%. Table A.7 gives the chi-squared critical value for 20 d.f. as 10.851, so the bound is $\dfrac{10.851}{2(550.87)} = .0098$. We can be 95% confident that l exceeds .0098.

b. Arguing as in **a**, $P\left(2l\Sigma X_i < c^2_{.05,2n}\right) = .95$. The following inequalities are equivalent to the one in parentheses:

$$l < \frac{c^2_{.05,2n}}{2\Sigma X_i} \Rightarrow -lt < \frac{-tc^2_{.05,2n}}{2\Sigma X_i} \Rightarrow e^{-lt} < \exp\left[\frac{-tc^2_{.05,2n}}{2\Sigma X_i}\right].$$

Replacing the ΣX_i by Σx_i in the expression on the right hand side of the last inequality gives a 95% lower confidence bound for e^{-lt}. Substituting t = 100, $c^2_{.05,20} = 31.410$ and $\Sigma x_i = 550.87$ gives .058 as the lower bound for the probability that time until breakdown exceeds 100 minutes.