CHAPTER 3

Section 3.1

1.

S:	FFF	SFF	FSF	FFS	FSS	SFS	SSF	SSS
X:	0	1	1	1	2	2	2	3

2. X = 1 if a randomly selected book is non-fiction and X = 0 otherwise
X = 1 if a randomly selected executive is a female and X = 0 otherwise
X = 1 if a randomly selected driver has automobile insurance and X = 0 otherwise

3. M = the difference between the large and the smaller outcome with possible values 0, 1, 2, 3, 4, or 5; W = 1 if the sum of the two resulting numbers is even and W = 0 otherwise, a Bernoulli random variable.

4. In my perusal of a zip code directory, I found no 00000, nor did I find any zip codes with four zeros, a fact which was not obvious. Thus possible X values are 2, 3, 4, 5 (and not 0 or 1). X = 5 for the outcome 15213, X = 4 for the outcome 44074, and X = 3 for 94322.

5. No. In the experiment in which a coin is tossed repeatedly until a H results, let Y = 1 if the experiment terminates with at most 5 tosses and Y = 0 otherwise. The sample space is infinite, yet Y has only two possible values.

6. Possible X values are1, 2, 3, 4, … (all positive integers)

Outcome:	RL	AL	RAARL	RRRRL	AARRL
X:	2	2	5	5	5

7.

a. Possible values are 0, 1, 2, …, 12; discrete

b. With N = # on the list, values are 0, 1, 2, … , N; discrete

c. Possible values are 1, 2, 3, 4, … ; discrete

d. $\{ x: 0 < x < \infty \}$ if we assume that a rattlesnake can be arbitrarily short or long; not discrete

e. With c = amount earned per book sold, possible values are 0, c, 2c, 3c, … , 10,000c; discrete

f. $\{ y: 0 < y < 14\}$ since 0 is the smallest possible pH and 14 is the largest possible pH; not discrete

g. With m and M denoting the minimum and maximum possible tension, respectively, possible values are $\{ x: m < x < M \}$; not discrete

h. Possible values are 3, 6, 9, 12, 15, … -- i.e. 3(1), 3(2), 3(3), 3(4), …giving a first element, etc,; discrete

8. Y = 3 : SSS; Y = 4: FSSS; Y = 5: FFSSS, SFSSS;
Y = 6: SSFSSS, SFFSSS, FSFSSS, FFFSSS;
Y = 7: SSFFS, SFSFSSS, SFFFSSS, FSSFSSS, FSFFSSS, FFSFSSS, FFFFSSS

9.

a. Returns to 0 can occur only after an even number of tosses; possible S values are 2, 4, 6, 8, …(i.e. 2(1), 2(2), 2(3), 2(4),…) an infinite sequence, so x is discrete.

b. Now a return to 0 is possible after any number of tosses greater than 1, so possible values are 2, 3, 4, 5, … (1+1,1+2, 1+3, 1+4, …, an infinite sequence) and X is discrete

10.

a. T = total number of pumps in use at both stations. Possible values: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

b. X: -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6

c. U: 0, 1, 2, 3, 4, 5, 6

d. Z: 0, 1, 2

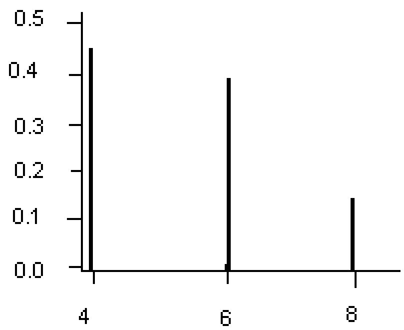
Section 3.2

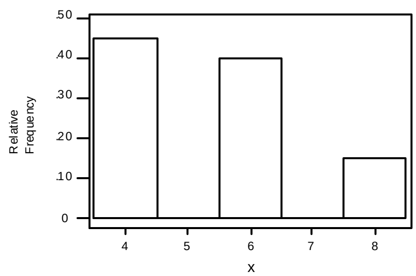
11.

a.

x	4	6	8
P(x)	.45	.40	.15

b.





c. P(x = 6) = .40 + .15 = .55 P(x > 6) = .15

12.

a. In order for the flight to accommodate all the ticketed passengers who show up, no more than 50 can show up. We need y = 50.
P(y = 50) = .05 + .10 + .12 + .14 + .25 + .17 = .83

b. Using the information in a. above, P(y > 50) = 1 - P(y = 50) = 1 - .83 = .17

c. For you to get on the flight, at most 49 of the ticketed passengers must show up. P(y = 49) = .05 + .10 + .12 + .14 + .25 = .66. For the 3rd person on the standby list, at most 47 of the ticketed passengers must show up. P(y = 44) = .05 + .10 + .12 = .27

13.

a. $P(X \le 3) = p(0) + p(1) + p(2) + p(3) = .10+.15+.20+.25 = .70$

b. $P(X < 3) = P(X \le 2) = p(0) + p(1) + p(2) = .45$

c. $P(3 \le X) = p(3) + p(4) + p(5) + p(6) = .55$

d. $P(2 \le X \le 5) = p(2) + p(3) + p(4) + p(5) = .71$

e. The number of lines not in use is 6 – X , so 6 – X = 2 is equivalent to X = 4, 6 – X = 3 to X = 3, and 6 – X = 4 to X = 2. Thus we desire $P(2 \le X \le 4) = p(2) + p(3) + p(4) = .65$

f. $6 - X \ge 4$ if $6 - 4 \ge X$, i.e. $2 \ge X$, or $X \le 2$, and $P(X \le 2) = .10+.15+.20 = .45$

14.

a. $\sum_{y=1}^{5} p(y)$ = K[1 + 2 + 3 + 4 + 5] = 15K = 1 $\Rightarrow K = \frac{1}{15}$

b. $P(Y \le 3) = p(1) + p(2) + p(3) = \frac{6}{15} = .4$

c. $P(2 \le Y \le 4) = p(2) + p(3) + p(4) = \frac{9}{15} = .6$

d. $\sum_{y=1}^{5}\left(\frac{y^2}{50}\right) = \frac{1}{50}[1+4+9+16+25] = \frac{55}{50} \ne 1$; No

15.

a. (1,2) (1,3) (1,4) (1,5) (2,3) (2,4) (2,5) (3,4) (3,5) (4,5)

b. $P(X = 0) = p(0) = P[\{ (3,4)\ (3,5)\ (4,5)\}] = \frac{3}{10} = .3$
$P(X = 2) = p(2) = P[\{ (1,2) \}] = \frac{1}{10} = .1$
$P(X = 1) = p(1) = 1 - [p(0) + p(2)] = .60$, and $p(x) = 0$ if $x \ne 0, 1, 2$

c. $F(0) = P(X \le 0) = P(X = 0) = .30$
$F(1) = P(X \le 1) = P(X = 0 \text{ or } 1) = .90$
$F(2) = P(X \le 2) = 1$

The c.d.f. is

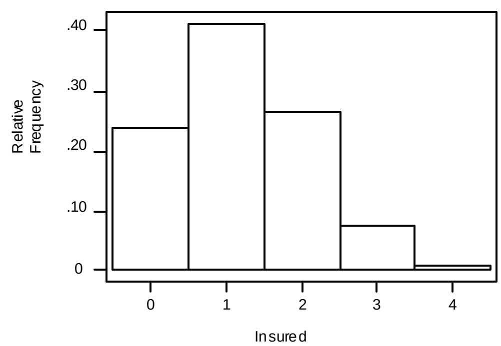
$$F(x) = \begin{cases} 0 & x < 0 \\ .30 & 0 \le x < 1 \\ .90 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

16.

a.

x	Outcomes	p(x)	
0	FFFF	$(.7)^4$	=.2401
1	FFFS,FFSF,FSFF,SFFF	$4[(.7)^3(.3)]$	=.4116
2	FFSS,FSFS,SFFS,FSSF,SFSF,SSFF	$6[(.7)^2(.3)^2]$	=.2646
3	FSSS, SFSS,SSFS,SSSF	$4[(.7)(.3)^3]$	=.0756
4	SSSS	$(.3)^4$	=.0081

b.



c. p(x) is largest for X = 1

d. $P(X \geq 2) = p(2) + p(3) + p(4) = .2646+.0756+.0081 = .3483$
This could also be done using the complement.

17.

a. $P(2) = P(Y = 2) = P(1^{st}$ 2 batteries are acceptable)
$= P(AA) = (.9)(.9) = .81$

b. $p(3) = P(Y = 3) = P(UAA \text{ or } AUA) = (.1)(.9)^2 + (.1)(.9)^2 = 2[(.1)(.9)^2] = .162$

c. The fifth battery must be an A, and one of the first four must also be an A. Thus, p(5) = P(AUUUA or UAUUA or UUAUA or UUUAA) $= 4[(.1)^3(.9)^2] = .00324$

d. $P(Y = y) = p(y) =$ P(the y^{th} is an A and so is exactly one of the first y – 1)
$=(y - 1)(.1)^{y-2}(.9)^2,\ y = 2,3,4,5,\ldots$

18.

a. p(1) = P(M = 1) = P[(1,1)] = $\frac{1}{36}$

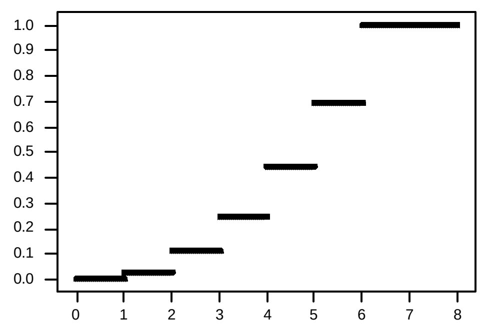
p(2) = P(M = 2) = P[(1,2) or (2,1) or (2,2)] = $\frac{3}{36}$

p(3) = P(M = 3) = P[(1,3) or (2,3) or (3,1) or (3,2) or (3,3)] = $\frac{5}{36}$

Similarly, p(4) = $\frac{7}{36}$, p(5) = $\frac{9}{36}$, and p(6) = $\frac{11}{36}$

b. F(m) = 0 for m < 1, $\frac{1}{36}$ for 1 ≤ m < 2,

$$F(m) = \begin{cases} 0 & m < 1 \\ \frac{1}{36} & 1 \le m < 2 \\ \frac{4}{36} & 2 \le m < 3 \\ \frac{9}{36} & 3 \le m < 4 \\ \frac{16}{36} & 4 \le m < 5 \\ \frac{25}{36} & 5 \le m < 6 \\ 1 & m \ge 6 \end{cases}$$



19. Let A denote the type O+ individual (type O positive blood) and B, C, D, the other 3 individuals. Then p(1) – P(Y = 1) = P(A first) = $\frac{1}{4}$ = .25

p(2) = P(Y = 2) = P(B, C, or D first and A next) = $\frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4} = .25$

p(4) = P(Y = 3) = P(A last) = $\frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{4} = .25$

So p(3) = 1 – (.25+.25+.25) = .25

20. P(0) = P(Y = 0) = P(both arrive on Wed.) = (.3)(.3) = .09
P(1) = P(Y = 1) = P[(W,Th)or(Th,W)or(Th,Th)]
= (.3)(.4) + (.4)(.3) + (.4)(.4) = .40
P(2) = P(Y = 2) = P[(W,F)or(Th,F)or(F,W) or (F,Th) or (F,F)] = .32
P(3) = 1 – [.09 + .40 + .32] = .19

21. The jumps in F(x) occur at x = 0, 1, 2, 3, 4, 5, and 6, so we first calculate F( ) at each of these values:

$F(0) = P(X \le 0) = P(X = 0) = .10$
$F(1) = P(X \le 1) = p(0) + p(1) = .25$
$F(2) = P(X \le 2) = p(0) + p(1) + p(2) = .45$
$F(3) = .70$, $F(4) = .90$, $F(5) = .96$, and $F(6) = 1$.

The c.d.f. is

$$F(x) = \begin{cases} .00 & x < 0 \\ .10 & 0 \le x < 1 \\ .25 & 1 \le x < 2 \\ .45 & 2 \le x < 3 \\ .70 & 3 \le x < 4 \\ .90 & 4 \le x < 5 \\ .96 & 5 \le x < 6 \\ 1.00 & 6 \le x \end{cases}$$

Then $P(X \le 3) = F(3) = .70$, $P(X < 3) = P(X \le 2) = F(2) = .45$,
$P(3 \le X) = 1 - P(X \le 2) = 1 - F(2) = 1 - .45 = .55$,
and $P(2 \le X \le 5) = F(5) - F(1) = .96 - .25 = .71$

22.

a. $P(X = 2) = .39 - .19 = .20$

b. $P(X > 3) = 1 - .67 = .33$

c. $P(2 \le X \le 5) = .92 - .19 = .78$

d. $P(2 < X < 5) = .92 - .39 = .53$

23.

a. Possible X values are those values at which F(x) jumps, and the probability of any particular value is the size of the jump at that value. Thus we have:

x	1	3	4	6	12
p(x)	.30	.10	.05	.15	.40

b. $P(3 \le X \le 6) = F(6) - F(3-) = .60 - .30 = .30$
$P(4 \le X) = 1 - P(X < 4) = 1 - F(4-) = 1 - .40 = .60$

24. $P(0) = P(Y = 0) = P(\text{B first}) = p$
$P(1) = P(Y = 1) = P(\text{G first, then B}) = P(GB) = (1 - p)p$
$P(2) = P(Y = 2) = P(GGB) = (1 - p)^2p$
Continuing, $p(y) = P(Y=y) = P(\text{y G's and then a B}) = (1 - p)^yp$ for y = 0,1,2,3,…

25.

a. Possible X values are 1, 2, 3, …

P(1) = P(X = 1) = P(return home after just one visit) = $\frac{1}{3}$

P(2) = P(X = 2) = P(second visit and then return home) = $\frac{2}{3} \cdot \frac{1}{3}$

P(3) = P(X = 3) = P(three visits and then return home) = $\left(\frac{2}{3}\right)^2 \cdot \frac{1}{3}$

In general p(x) = $\left(\frac{2}{3}\right)^{x-1}\left(\frac{1}{3}\right)$ for x = 1, 2, 3, …

b. The number of straight line segments is Y = 1 + X (since the last segment traversed returns Alvie to O), so as in a, p(y) = $\left(\frac{2}{3}\right)^{y-2}\left(\frac{1}{3}\right)$ for y = 2, 3, …

c. Possible Z values are 0, 1, 2, 3 , …

p(0) = P(male first and then home) = $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$,

p(1) = P(exactly one visit to a female) = P(female 1^{st}, then home) + P(F, M, home) + P(M, F, home) + P(M, F, M, home)

$= \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$

$= \left(\frac{1}{2}\right)\left(1 + \frac{2}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3} + 1\right)\left(\frac{1}{3}\right) = \left(\frac{1}{2}\right)\left(\frac{5}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)\left(\frac{1}{3}\right)$

where the first term corresponds to initially visiting a female and the second term corresponds to initially visiting a male. Similarly,

p(2) = $\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^2\left(\frac{5}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^2\left(\frac{5}{3}\right)\left(\frac{1}{3}\right)$. In general,

p(z) = $\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^{2z-2}\left(\frac{5}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^{2z-2}\left(\frac{5}{3}\right)\left(\frac{1}{3}\right) = \left(\frac{24}{54}\right)\left(\frac{2}{3}\right)^{2z-2}$ for z = 1, 2, 3, …

26.

a. The sample space consists of all possible permutations of the four numbers 1, 2, 3, 4:

outcome	y value	outcome	y value	outcome	y value
1234	4	2314	1	3412	0
1243	2	2341	0	3421	0
1324	2	2413	0	4132	1
1342	1	2431	1	4123	0
1423	1	3124	1	4213	1
1432	2	3142	0	4231	2
2134	2	3214	2	4312	0
2143	0	3241	1	4321	0

b. Thus p(0) = P(Y = 0) = $\frac{9}{24}$, p(1) = P(Y = 1) = $\frac{8}{24}$, p(2) = P(Y = 2) = $\frac{6}{24}$,

p(3) = P(Y = 3) = 0, p(3) = P(Y = 3) = $\frac{1}{24}$.

27. If $x_1 < x_2$, $F(x_2) = P(X \le x_2) = P(\{X \le x_1\} \cup \{x_1 < X \le x_2\})$
$= P(X \le x_1) + P(x_1 < X \le x_2) \ge P(X \le x_1) = F(x_1)$.
$F(x_1) = F(x_2)$ when $P(x_1 < X \le x_2) = 0$.

Section 3.3

28.

a. $E(X) = \sum_{x=0}^{4} x \cdot p(x)$
$= (0)(.08) + (1)(.15) + (2)(.45) + (3)(.27) + (4)(.05) = 2.06$

b. $V(X) = \sum_{x=0}^{4} (x - 2.06)^2 \cdot p(x) = (0 - 2.06)^2(.08) + \ldots + (4 - 2.06)^2(.05)$
$= .339488 + .168540 + .001620 + .238572 + .188180 = .9364$

c. $\sigma_x = \sqrt{.9364} = .9677$

d. $V(X) = \left[\sum_{x=0}^{4} x^2 \cdot p(x)\right] - (2.06)^2 = 5.1800 - 4.2436 = .9364$

29.

a. $E(Y) = \sum_{x=0}^{4} y \cdot p(y) = (0)(.60) + (1)(.25) + (2)(.10) + (3)(.05) = .60$

b. $E(100Y^2) = \sum_{x=0}^{4} 100y^2 \cdot p(y) = (0)(.60) + (100)(.25)$
$+ (400)(.10) + (900)(.05) = 110$

30. $E(Y) = .60$;
$E(Y^2) = 1.1$
$V(Y) = E(Y^2) - [E(Y)]^2 = 1.1 - (.60)^2 = .74$
$\sigma_y = \sqrt{.74} = .8602$
$E(Y) \pm \sigma_y = .60 \pm .8602 = (-.2602, 1.4602)$ or $(0, 1)$.
$P(Y = 0) + P(Y = 1) = .85$

31.

a. $E(X) = (13.5)(.2) + (15.9)(.5) + (19.1)(.3) = 16.38,$
$E(X^2) = (13.5)^2(.2) + (15.9)^2(.5) + (19.1)^2(.3) = 272.298,$
$V(X) = 272.298 - (16.38)^2 = 3.9936$

b. $E(25X - 8.5) = 25\,E(X) - 8.5 = (25)(16.38) - 8.5 = 401$

c. $V(25X - 8.5) = V(25X) = (25)^2V(X) = (625)(3.9936) = 2496$

d. $E[h(X)] = E[X - .01X^2] = E(X) - .01E(X^2) = 16.38 - 2.72 = 13.66$

32.

a. $E(X^2) = \sum_{x=0}^{1} x^2 \cdot p(x) = (0^2)((1-p) + (1^2)(p) = (1)(p) = p$

b. $V(X) = E(X^2) - [E(X)]^2 = p - p^2 = p(1-p)$

c. $E(x^{79}) = (0^{79})(1-p) + (1^{79})(p) = p$

33. $E(X) = \sum_{x=1}^{\infty} x \cdot p(x) = \sum_{x=1}^{\infty} x \cdot \frac{c}{x^3} = c\sum_{x=1}^{\infty}\frac{1}{x^2}$, but it is a well-known result from the theory of infinite series that $\sum_{x=1}^{\infty}\frac{1}{x^2} < \infty$, so E(X) is finite.

34. Let h(X) denote the net revenue (sales revenue – order cost) as a function of X. Then $h_3(X)$ and $h_4(X)$ are the net revenue for 3 and 4 copies purchased, respectively. For x = 1 or 2, $h_3(X) = 2x - 3$, but at x = 3,4,5,6 the revenue plateaus. Following similar reasoning, $h_4(X) = 2x - 4$ for x=1,2,3, but plateaus at 4 for x = 4,5,6.

x	1	2	3	4	5	6
$h_3(x)$	-1	1	3	3	3	3
$h_4(x)$	-2	0	2	4	4	4
p(x)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$

$E[h_3(X)] = \sum_{x=1}^{6} h_3(x) \cdot p(x) = (-1)(\frac{1}{15}) + \ldots + (3)(\frac{2}{15}) = 2.4667$

Similarly, $E[h_4(X)] = \sum_{x=1}^{6} h_4(x) \cdot p(x) = (-2)(\frac{1}{15}) + \ldots + (4)(\frac{2}{15}) = 2.6667$

Ordering 4 copies gives slightly higher revenue, on the average.

35.

P(x)	.8	.1	.08	.02
x	0	1,000	5,000	10,000
H(x)	0	500	4,500	9,500

E[h(X)] = 600. Premium should be \$100 plus expected value of damage minus deductible or \$700.

36. $E(X) = \sum_{x=1}^{n} x \cdot \left(\frac{1}{n}\right) = \left(\frac{1}{n}\right)\sum_{x=1}^{n} x = \frac{1}{n}\left[\frac{n(n+1)}{2}\right] = \frac{n+1}{2}$

$E(X^2) = \sum_{x=1}^{n} x^2 \cdot \left(\frac{1}{n}\right) = \left(\frac{1}{n}\right)\sum_{x=1}^{n} x^2 = \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6}\right] = \frac{(n+1)(2n+1)}{6}$

So $V(X) = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{n^2-1}{12}$

37. $E[h(X)] = E\left(\frac{1}{X}\right) = \sum_{x=1}^{6}\left(\frac{1}{x}\right) \cdot p(x) = \frac{1}{6}\sum_{x=1}^{6}\frac{1}{x} = .408$, whereas $\frac{1}{3.5} = .286$, so you expect to win more if you gamble.

38. $E(X) = \sum_{x=1}^{4} x \cdot p(x) = 2.3$, $E(X^2) = 6.1$, so $V(X) = 6.1 - (2.3)^2 = .81$

Each lot weighs 5 lbs, so weight left = 100 – 5x.
Thus the expected weight left is 100 – 5E(X) = 88.5,
and the variance of the weight left is
V(100 – 5X) = V(-5X) = 25V(x) = 20.25.

39.

a. The line graph of the p.m.f. of –X is just the line graph of the p.m.f. of X reflected about zero, but both have the same degree of spread about their respective means, suggesting V(-X) = V(X).

b. With a = -1, b = 0, $V(aX + b) = V(-X) = a^2V(X)$.

40. $V(aX + b) = \sum_x [aX + b - E(aX+b)]^2 \cdot p(x) = \sum_x [aX + b - (am + b)]^2 p(x)$

$$= \sum_x [aX - (am)]^2 p(x) = a^2 \sum_x [X - m]^2 p(x) = a^2 V(X).$$

41.

a. $E[X(X-1)] = E(X^2) - E(X)$, $\Rightarrow E(X^2) = E[X(X-1)] + E(X) = 32.5$

b. $V(X) = 32.5 - (5)^2 = 7.5$

c. $V(X) = E[X(X-1)] + E(X) - [E(X)]^2$

42. With a = 1 and b = c, E(X – c) = E(aX + b) = aE(X) + b = E(X) – c. When c = μ, E(X - μ) = E(X) - μ = μ - μ = 0, so the expected deviation from the mean is zero.

43.

a.

k	2	3	4	5	10
$\frac{1}{k^2}$	.25	.11	.06	.04	.01

b. $m = \sum_{x=0}^{6} x \cdot p(x) = 2.64$, $s^2 = \left[\sum_{x=0}^{6} x^2 \cdot p(x)\right] - m^2 = 2.37,\ s = 1.54$

Thus μ - 2σ = -.44, and μ + 2σ = 5.72,
so P(|x-μ| ≥ 2σ) = P(X is lat least 2 s.d.'s from μ)
= P(x is either ≤-.44 or ≥ 5.72) = P(X = 6) = .04.
Chebyshev's bound of .025 is much too conservative. For K = 3,4,5, and 10, P(|x-μ| ≥ kσ) = 0, here again pointing to the very conservative nature of the bound $\frac{1}{k^2}$.

c. μ = 0 and $s = \frac{1}{3}$, so P(|x-μ| ≥ 3σ) = P(| X | ≥ 1)
= P(X = -1 or +1) = $\frac{1}{18} + \frac{1}{18} = \frac{1}{9}$, identical to the upper bound.

d. Let p(-1) = $\frac{1}{50}$, $p(+1) = \frac{1}{50}$, $p(0) = \frac{24}{25}$.

Section 3.4

44.

a. b(3;8,.6) = $\binom{8}{3}(.6)^3(.4)^5$ = (56)(.00221184) = .124

b. b(5;8,.6) = $\binom{8}{5}(.6)^5(.4)^3$ = (56)(.00497664) = .279

c. P(3 ≤ X ≤ 5) = b(3;8,.6) + b(4;8,.6) + b(5;8,.6) = .635

d. P(1 ≤ X) = 1 – P(X = 0) = 1 - $\binom{12}{0}(.1)^0(.9)^{12}$ = 1 – $(.9)^{12}$ = .718

45.

a. B(4;10,.3) = .850

b. b(4;10,.3) = B(4;10,.3) - B(3;10,.3) = .200

c. b(6;10,.7) = B(6;10,.7) - B(5;10,.7) = .200

d. P(2 ≤ X ≤ 4) = B(4;10,.3) - B(1;10,.3) = .701

e. P(2 < X) = 1 - P(X ≤ 1) = 1 - B(1;10,.3) = .851

f. P(X ≤ 1) = B(1;10,.7) = .0000

g. P(2 < X < 6) = P(3 ≤ X ≤ 5) = B(5;10,.3) - B(2;10,.3) = .570

46. X ~ Bin(25, .05)

a. P(X ≤ 2) = B(2;25,.05) = .873

b. P(X ≥ 5) = 1 - P(X ≤ 4) = 1 – B(4;25,.05) = .1 - .993 = .007

c. P(1 ≤ X ≤ 4) = P(X ≤ 4) – P(X ≤ 0) = .993 - .277 = .716

d. P(X = 0) = P(X ≤ 0) = .277

e. E(X) = np = (25)(.05) = 1.25
V(X) = np(1 – p) = (25)(.05)(.95) =1.1875
σ_x = 1.0897

47. X ~ Bin(6, .10)

a. P(X = 1) = $\binom{n}{x}(p)^x(1-p)^{n-x}$ = $\binom{6}{1}(.1)^1(.9)^5 = .3543$

b. P(X ≥ 2) = 1 – [P(X = 0) + P(X = 1)].

From **a** , we know P(X = 1) = .3543, and P(X = 0) = $\binom{6}{0}(.1)^0(.9)^6 = .5314$.

Hence P(X ≥ 2) = 1 – [.3543 + .5314] = .1143

c. Either 4 or 5 goblets must be selected

i) Select 4 goblets with zero defects: P(X = 0) = $\binom{4}{0}(.1)^0(.9)^4 = .6561$.

ii) Select 4 goblets, one of which has a defect, and the 5th is good:

$$\left[\binom{4}{1}(.1)^1(.9)^3\right]\times .9 = .26244$$

So the desired probability is .6561 + .26244 = .91854

48. Let S = comes to a complete stop, so p = .25 , n = 20

a. P(X ≤ 6) = B(6;20,.25) = .786

b. P(X = 6) = b(6;20,.20) = B(6;20,.25) - B(5;20,.25) = .786 - .617 = .169

c. P(X ≥ 6) = 1 – P(X ≤ 5) = 1 - B(5;20,.25) = 1 - .617 = .383

d. E(X) = (20)(.25) = 5. We expect 5 of the next 20 to stop.

49. Let S = has at least one citation. Then p = .4, n = 15

a. If at least 10 have no citations (Failure), then at most 5 have had at least one (Success):

P(X ≤ 5) = B(5;15,.40) = .403

b. P(X ≤ 7) = B(7;15,.40) = .787

c. P(5 ≤ X ≤ 10) = P(X ≤ 10) – P(X ≤ 4) = .991 - .217 = .774

50. $X \sim \text{Bin}(10, .60)$

a. $P(X \geq 6) = 1 - P(X \leq 5) = 1 - B(5;20,.60) = 1 - .367 = .633$

b. $E(X) = np = (10)(.6) = 6$; $V(X) = np(1-p) = (10)(.6)(.4) = 2.4$;
$\sigma_x = 1.55$
$E(X) \pm \sigma_x = (\ 4.45, 7.55\ )$.
We desire $P(\ 5 \leq X \leq 7) = P(X \leq 7) - P(X \leq 4) = .833 - .166 = .667$

c. $P(\ 3 \leq X \leq 7) = P(X \leq 7) - P(X \leq 2) = .833 - .012 = .821$

51. Let S represent a telephone that is submitted for service while under warranty and must be replaced. Then $p = P(S) = P(\text{replaced} \mid \text{submitted}) \cdot P(\text{submitted}) = (.40)(.20) = .08$. Thus X, the number among the company's 10 phones that must be replaced, has a binomial distribution with n = 10, p = .08, so $p(2) = P(X=2) = \binom{10}{2}(.08)^2(.92)^8 = .1478$

52. $X \sim \text{Bin}\ (25, .02)$

a. $P(X=1) = 25(.02)(.98)^{24} = .308$

b. $P(X=1) = 1 - P(X=0) = 1 - (.98)^{25} = 1 - .603 = .397$

c. $P(X=2) = 1 - P(X=1) = 1 - [.308 + .397]$

d. $\bar{x} = 25(.02) = .5$; $s = \sqrt{npq} = \sqrt{25(.02)(.98)} = \sqrt{.49} = .7$
$\bar{x} + 2s = .5 + 1.4 = 1.9$ So $P(0 = X = 1.9 = P(X=1) = .705$

e. $\dfrac{.5(4.5) + 24.5(3)}{25} = 3.03$ hours

53. X = the number of flashlights that work.
Let event B = {battery has acceptable voltage}.
Then P(flashlight works) = P(both batteries work) = P(B)P(B) = (.9)(.9) = .81 We must assume that the batteries' voltage levels are independent.
$X \sim \text{Bin}\ (10, .81)$. $P(X=9) = P(X=9) + P(X=10)$

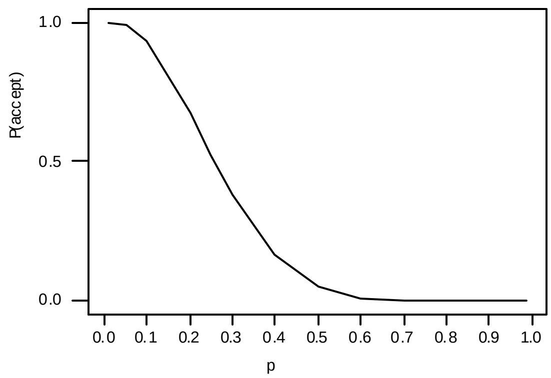
$$\binom{10}{9}(.81)^9(.19) + \binom{10}{10}(.81)^{10} = .285 + .122 = .407$$

54. Let p denote the actual proportion of defectives in the batch, and X denote the number of defectives in the sample.

a. P(the batch is accepted) = $P(X \le 2) = B(2;10,p)$

p	.01	.05	.10	.20	.25
P(accept)	1.00	.988	.930	.678	.526

b.



c. P(the batch is accepted) = $P(X \le 1) = B(1;10,p)$

p	.01	.05	.10	.20	.25
P(accept)	.996	.914	.736	.376	.244

d. P(the batch is accepted) = $P(X \le 2) = B(2;15,p)$

p	.01	.05	.10	.20	.25
P(accept)	1.00	.964	.816	.398	.236

e. We want a plan for which P(accept) is high for $p \le .1$ and low for $p > .1$
The plan in **d** seems most satisfactory in these respects.

55.

a. P(rejecting claim when p = .8) = B(15;25,.8) = .017

b. P(not rejecting claim when p = .7) = P(X ≥ 16 when p = .7)
= 1 - B(15;25,.7) = 1 - .189 = .811; for p = .6, this probability is
= 1 - B(15;25,.6) = 1 - .575 = .425.

c. The probability of rejecting the claim when p = .8 becomes B(14;25,.8) = .006, smaller than in **a** above. However, the probabilities of **b** above increase to .902 and .586, respectively.

56. h(x) = 1 · X + 2.25(25 – X) = 62.5 – 1.5X, so E(h(X)) = 62.5 – 1.5E(x)
= 62.5 – 1.5np – 62.5 – (1.5)(25)(.6) = $40.00

57. If topic A is chosen, when n = 2, P(at least half received)
= P(X ≥ 1) = 1 – P(X = 0) = 1 – $(.1)^2$ = .99
If B is chosen, when n = 4, P(at least half received)
= P(X ≥ 2) = 1 – P(X ≤ 1) = 1 – $(0.1)^4 - 4(.1)^3(.9)$ = .9963
Thus topic B should be chosen.
If p = .5, the probabilities are .75 for A and .6875 for B, so now A should be chosen.

58.

a. np(1 – p) = 0 if either p = 0 (whence every trial is a failure, so there is no variability in X) or if p = 1 (whence every trial is a success and again there is no variability in X)

b. $\frac{d}{dp}\left[np(1-p)\right]$ = n[(1 – p) + p(-1)] = n[1 – 2p = 0 ⇒ p = .5, which is easily seen to correspond to a maximum value of V(X).

59.

a. b(x; n, 1 – p) = $\binom{n}{x}(1-p)^x(p)^{n-x} = \binom{n}{n-x}(p)^{n-x}(1-p)^x$ = b(n-x; n, p)
Alternatively, P(x S's when P(S) = 1 – p) = P(n-x F's when P(F) = p), since the two events are identical), but the labels S and F are arbitrary so can be interchanged (if P(S) and P(F) are also interchanged), yielding P(n-x S's when P(S) = 1 – p) as desired.

b. B(x;n,1 – p) = P(at most x S's when P(S) = 1 – p)
= P(at least n-x F's when P(F) = p)
= P(at least n-x S's when P(S) = p)
= 1 – P(at most n-x-1 S's when P(S) = p)
= 1 – B(n-x-1;n,p)

c. Whenever p > .5, (1 – p) < .5 so probabilities involving X can be calculated using the results **a** and **b** in combination with tables giving probabilities only for p ≤ .5

60. Proof of E(X) = np:

$$E(X) = \sum_{x=0}^{n} x \cdot \binom{n}{x} p^x (1-p)^{n-x} = \sum_{x=1}^{n} x \cdot \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x} = np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$$

$$= np \sum_{y=0}^{n} \frac{(n-1)!}{(y)!(n-1-y)!} p^y (1-p)^{n-1-y} \quad \text{(y replaces x-1)}$$

$$= np \left\{ \sum_{y=0}^{n-1} \binom{n-1}{y} p^y (1-p)^{n-1-y} \right\}$$

The expression in braces is the sum over all possible values y = 0, 1, 2, … , n-1 of a binomial p.m.f. based on n-1 trials, so equals 1, leaving only np, as desired.

61.

a. Although there are three payment methods, we are only concerned with S = uses a debit card and F = does not use a debit card. Thus we can use the binomial distribution. So n = 100 and p = .5. E(X) = np = 100(.5) = 50, and V(X) = 25.

b. With S = doesn't pay with cash, n = 100 and p = .7, E(X) = np = 100(.7) = 70, and V(X) = 21.

62.

a. Let X = the number with reservations who show, a binomial r.v. with n = 6 and p = .8. The desired probability is
P(X = 5 or 6) = b(5;6,.8) + b(6;6,.8) = .3932 + .2621 = .6553

b. Let h(X) = the number of available spaces. Then

When x is:	0	1	2	3	4	5	6
H(x) is:	4	3	2	1	0	0	0

E[h(X)] = $\sum_{x=0}^{6} h(x) \cdot b(x;6,.8)$ = 4(.000) + 3(.002) = 2(.015 + 3(.082) = .277

c. Possible X values are 0, 1, 2, 3, and 4. X = 0 if there are 3 reservations and none show or …or 6 reservations and none show, so
P(X = 0) = b(0;3,.8)(.1) + b(0;4,.8)(.2) + b(0;5,.8)(.3) + b(0;6,.8)(.4)
= .0080(.1) + .0016(.2) + .0003(.3) + .0001(.4) = .0013
P(X = 1) = b(1;3,.8)(.1) + … + b(1;6,.8)(.4) = .0172
P(X = 2) = .0906, P(X = 3) = .2273,
P(X = 4) = 1 – [.0013 + … + .2273] = .6636

63. When p = .5, μ = 10 and σ = 2.236, so 2σ = 4.472 and 3σ = 6.708.
The inequality $|X - 10| \geq 4.472$ is satisfied if either $X \leq 5$ or $X \geq 15$, or $P(|X - \mu| \geq 2\sigma) = P(X \leq 5 \text{ or } X \geq 15) = .021 + .021 = .042$.

In the case p = .75, μ = 15 and σ = 1.937, so 2σ = 3.874 and 3σ = 5.811. $P(|X - 15| \geq 3.874) = P(X \leq 11 \text{ or } X \geq 19) = .041 + .024 = .065$, whereas $P(|X - 15| \geq 5.811) = P(X \leq 9) = .004$. All these probabilities are considerably less than the upper bounds .25(for k = 2) and .11 (for k = 3) given by Chebyshev.

Section 3.5

64.

a. X ~ Hypergeometric N=15, n=5, M=6

b. P(X=2) = $\dfrac{\binom{6}{2}\binom{9}{3}}{\binom{15}{5}} = \dfrac{840}{3003} = .280$

P(X=2) = P(X=0) + P(X=1) + P(X=2)

$$= \frac{\binom{9}{5}}{\binom{15}{5}} + \frac{\binom{6}{1}\binom{9}{4}}{\binom{15}{5}} + \frac{840}{3003} = \frac{126+756+840}{3003} = \frac{1722}{3003} = .573$$

P(X=2) = 1 – P(X=1) = 1 – [P(X=0) + P(X=1)] = $1 - \dfrac{126+756}{3003} = .706$

c. E(X) = $5\left(\dfrac{6}{15}\right) = 2$; V(X) = $\left(\dfrac{15-5}{14}\right)\cdot 5 \cdot \left(\dfrac{6}{15}\right)\cdot\left(1-\dfrac{6}{15}\right) = .857$;
$s = \sqrt{V(X)} = .926$

65. X~h(x; 6, 12, 7)

a. P(X=5) = $\dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} = \dfrac{105}{924} = .114$

b. P(X=4) = 1 – P(X=5) = 1 – [P(X=5) + P(X=6)] =

$$1-\left[\frac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}}+\frac{\binom{7}{6}}{\binom{12}{6}}\right]=1-\frac{105+7}{924}=1-.121=.879$$

c. E(X) = $\left(\dfrac{6\cdot 7}{12}\right)=3.5$; $s = \sqrt{\left(\frac{6}{11}\right)(6)\left(\frac{7}{12}\right)\left(\frac{5}{12}\right)} = \sqrt{.795} = .892$

P(X > 3.5 + .892) = P(X > 4.392) = P(X=5) = .121 (see part b)

d. We can approximate the hypergeometric distribution with the binomial if the population size and the number of successes are large: h(x;15,40,400) approaches b(x;15,.10). So P(X=5) ˜ B(5; 15, .10) from the binomial tables = .998

66.

a. P(X = 10) = h(10;15,30,50) = $\dfrac{\binom{30}{10}\binom{20}{5}}{\binom{50}{15}} = .2070$

b. P(X ≥ 10) = h(10;15,30,50) + h(11;15,30,50) + … + h(15;15,30,50)
= .2070+.1176+.0438+.0101+.0013+.0001 = .3799

c. P(at least 10 from the same class) = P(at least 10 from second class [answer from **b**]) + P(at least 10 from first class). But "at least 10 from 1st class" is the same as "at most 5 from the second" or P(X ≤ 5).

P(X ≤ 5) = h(0;15,30,50) + h(1;15,30,50) + … + h(5;15,30,50)
= 11697+.002045+.000227+.000150+.000001+.000000
= .01412

So the desired probability = P(x ≥ 10) + P(X ≤ 5)
= .3799 + .01412 = .39402

d. E(X) = $n \cdot \frac{M}{N} = 15 \cdot \frac{30}{50} = 9$

V(X) = $\left(\frac{35}{49}\right)(9)\left(1 - \frac{30}{50}\right) = 2.5714$

σ_x = 1.6036

e. Let Y = 15 – X. Then E(Y) = 15 – E(X) = 15 – 9 = 6

V(Y) = V(15 – X) – V(X) = 2.5714, so σ_Y = 1.6036

67.

a. Possible values of X are 5, 6, 7, 8, 9, 10. (In order to have less than 5 of the granite, there would have to be more than 10 of the basaltic).

P(X = 5) = h(5; 15,10,20) = $\frac{\binom{10}{5}\binom{10}{10}}{\binom{20}{15}} = .0163$.

Following the same pattern for the other values, we arrive at the pmf, in table form below.

x	5	6	7	8	9	10
p(x)	.0163	.1354	.3483	.3483	.1354	.0163

b. P(all 10 of one kind or the other) = P(X = 5) + P(X = 10) = .0163 + .0163 = .0326

c. E(X) = $n \cdot \frac{M}{N} = 15 \cdot \frac{10}{20} = 7.5$; V(X) = $\left(\frac{5}{19}\right)(7.5)\left(1 - \frac{10}{20}\right) = .9868$;

σ_x = .9934

$\mu \pm \sigma$ = 7.5 ± .9934 = (6.5066, 8.4934), so we want

P(X = 7) + P(X = 8) = .3483 + .3483 = .6966

68.

a. h(x; 6,4,11)

b. $6 \cdot \left(\frac{4}{11}\right) = 2.18$

69.

a. h(x; 10,10,20) (the successes here are the top 10 pairs, and a sample of 10 pairs is drawn from among the 20)

b. Let X = the number among the top 5 who play E-W. Then P(all of top 5 play the same direction) = P(X = 5) + P(X = 0) = h(5;10,5,20) + h(5;10,5,20)

$$= \frac{\binom{15}{5}}{\binom{20}{10}} + \frac{\binom{15}{10}}{\binom{20}{10}} = .033$$

c. N = 2n; M = n; n = n
h(x;n,n,2n)
E(X) = $n \cdot \frac{n}{2n} = \frac{1}{2}n$;
V(X) =
$$\left(\frac{2n-n}{2n-1}\right) \cdot n \cdot \frac{n}{2n} \cdot \left(1 - \frac{n}{2n}\right) = \left(\frac{n}{2n-1}\right) \cdot \frac{n}{2} \cdot \left(1 - \frac{n}{2n}\right) = \left(\frac{n}{2n-1}\right) \cdot \frac{n}{2} \cdot \left(\frac{1}{2}\right)$$

70.

a. h(x;10,15,50)

b. When N is large relative to n, h(x; n,M,N) $\approx b\left(x; n, \frac{M}{N}\right)$,

so h(x;10,150,500) $\approx b(x;10,.3)$

c. Using the hypergeometric model, E(X) = $10 \cdot \left(\frac{150}{500}\right) = 3$ and

V(X) = $\frac{490}{499}(10)(.3)(.7) = .982(2.1) = 2.06$
Using the binomial model, E(X) = (10)(.3) = 3, and
V(X) = 10(.3)(.7) = 2.1

71.

a. With S = a female child and F = a male child, let X = the number of F's before the 2^{nd} S. Then P(X = x) = nb(x;2, .5)

b. P(exactly 4 children) = P(exactly 2 males)
= nb(2;2,.5) = (3)(.0625) = .188

c. P(at most 4 children) = P(X ≤ 2)

$$= \sum_{x=0}^{2} nb(x;2,.5) = .25+2(.25)(.5) + 3(.0625) = .688$$

d. E(X) = $\frac{(2)(.5)}{.5} = 2$, so the expected number of children = E(X + 2)
= E(X) + 2 = 4

72. The only possible values of X are 3, 4, and 5.
p(3) = P(X = 3) = P(first 3 are B's or first 3 are G's) = $2(.5)^3$ = .250
p(4) = P(two among the 1^{st} three are B's and the 4th is a B) + P(two among the 1^{st} three are G's and the 4th is a G) = $2 \cdot \binom{3}{2}(.5)^4 = .375$
p(5) = 1 – p(3) – p(4) = .375

73. This is identical to an experiment in which a single family has children until exactly 6 females have been born(since p = .5 for each of the three families), so p(x) = nb(x;6,.5) and E(X) = 6 (= 2+2+2, the sum of the expected number of males born to each one.)

74. The interpretation of "roll" here is a pair of tosses of a single player's die(two tosses by A or two by B). With S = doubles on a particular roll, p = $\frac{1}{6}$. Furthermore, A and B are really identical (each die is fair), so we can equivalently imagine A rolling until 10 doubles appear. The P(x rolls) = P(9 doubles among the first x – 1 rolls and a double on the x^{th} roll =

$$\binom{x-1}{9}\left(\frac{5}{6}\right)^{x-10}\left(\frac{1}{6}\right)^{9}\cdot\left(\frac{1}{6}\right)=\binom{x-1}{9}\left(\frac{5}{6}\right)^{x-10}\left(\frac{1}{6}\right)^{10}$$

E(X) = $\frac{r(1-p)}{p} = \frac{10(\frac{5}{6})}{\frac{1}{6}} = 10(5) = 50$ V(X) = $\frac{r(1-p)}{p^2} = \frac{10(\frac{5}{6})}{(\frac{1}{6})^2} = 10(5)(6) = 300$

Section 3.6

75.

a. $P(X \le 8) = F(8;5) = .932$

b. $P(X = 8) = F(8;5) - F(7;5) = .065$

c. $P(X \ge 9) = 1 – P(X \le 8) = .068$

d. $P(5 \le X \le 8) = F(8;5) – F(4;5) = .492$

e. $P(5 < X < 8) = F(7;5) – F(5;5) = .867-.616=.251$

76.

a. $P(X \le 5) = F(5;8) = .191$

b. $P(6 \le X \le 9) = F(9;8) – F(5;8) = .526$

c. $P(X \ge 10) = 1 – P(X \le 9) = .283$

d. $E(X) = \lambda= 10$, $\sigma_X = \sqrt{l} = 2.83$, so $P(X > 12.83) = P(X \ge 13) = 1 – P(X \le 12) =1 - .936 = .064$

77.

a. $P(X \le 10) = F(10;20) = .011$

b. $P(X > 20) = 1 – F(20;20) = 1 - .559 = .441$

c. $P(10 \le X \le 20) = F(20;20) – F(9;20) = .559 - .005 = .554$
$P(10 < X < 20) = F(19;20) – F(10;20) = .470 - .011 = .459$

d. $E(X) = \lambda= 20$, $\sigma_X = \sqrt{l} = 4.472$

$$
\begin{aligned}
P(\mu - 2\sigma < X < \mu + 2\sigma) &= P(20 – 8.944 < X < 20 + 8.944) \\
&= P(11.056 < X < 28.944) \\
&= P(X \le 28) - P(X \le 11) \\
&= F(28;20) - F(12;20)] \\
&= .966 - .021 = .945
\end{aligned}
$$

78.

a. $P(X = 1) = F(1;2) – F(0;2) = .982 - .819 = .163$

b. $P(X \ge 2) = 1 – P(X \le 1) = 1 – F(1;2) = 1 - .982 = .018$

c. $P(1^{st}\text{ doesn't} \cap 2^{nd}\text{ doesn't}) = P(1^{st}\text{ doesn't}) \cdot P(2^{nd}\text{ doesn't})$
$= (.819)(.819) = .671$

79. $p = \frac{1}{200}$; n = 1000; λ = np = 5

a. $P(5 \le X \le 8) = F(8;5) – F(4;5) = .492$

b. $P(X \ge 8) = 1 – P(X \le 7) = 1 - .867 = .133$

80.

a. The experiment is binomial with n = 10,000 and p = .001,
so μ = np = 10 and $\sigma = \sqrt{npq} = 3.161$.

b. X has approximately a Poisson distribution with λ = 10,
so P(X > 10) ˜ 1 – F(10;10) = 1 - .583 = .417

c. P(X = 0) ˜ 0

81.

a. λ = 8 when t = 1, so P(X = 6) = F(6;8) – F(5;8) =.313 - .191 = .122,
$P(X \ge 6) = 1 - F(5;8) = .809$, and $P(X \ge 10) = 1 - F(9;8) = .283$

b. t = 90 min = 1.5 hours, so λ = 12; thus the expected number of arrivals is 12 and the SD $= \sqrt{12} = 3.464$

c. t = 2.5 hours implies that λ = 20; in this case, $P(X \ge 20) = 1 – F(19;20) = .530$ and $P(X \le 10) = F(10;20) = .011$.

82.

a. P(X = 4) = F(4;5) – F(3;5) = .440 - .265 = .175

b. $P(X \ge 4) = 1 - P(X \le 3) = 1 - .265 = .735$

c. Arrivals occur at the rate of 5 per hour, so for a 45 minute period the rate is λ = (5)(.75) = 3.75, which is also the expected number of arrivals in a 45 minute period.

83.

a. For a two hour period the parameter of the distribution is λt = (4)(2) = 8,
so P(X = 10) = F(10;8) – F(9;8) = .099.

b. For a 30 minute period, λt = (4)(.5) = 2, so P(X = 0) = F(0,2) = .135

c. E(X) = λt = 2

84. Let X = the number of diodes on a board that fail.

a. $E(X) = np = (200)(.01) = 2$, $V(X) = npq = (200)(.01)(.99) = 1.98$, $\sigma_X = 1.407$

b. X has approximately a Poisson distribution with $\lambda = np = 2$,
so $P(X \ge 4) = 1 - P(X \le 3) = 1 - F(3;2) = 1 - .857 = .143$

c. P(board works properly) = P(all diodes work) = $P(X = 0) = F(0;2) = .135$
Let Y = the number among the five boards that work, a binomial r.v. with n = 5 and p = .135. Then $P(Y \ge 4) = P(Y = 4) + P(Y = 5) =$

$$\binom{5}{4}(.135)^4(.865) + \binom{5}{5}(.135)^5(.865)^0 = .00144 + .00004 = .00148$$

85. α = 1/(mean time between occurrences) = $\dfrac{1}{.5} = 2$

a. $\alpha t = (2)(2) = 4$

b. $P(X > 5)\ 1 - P(X \le 5) = 1 - .785 = .215$

c. Solve for t , given $\alpha = 2$:

$.1 = e^{-\alpha t}$

$\ln(.1) = -\alpha t$

$t = \dfrac{2.3026}{2} \approx 1.15$ years

86. $$E(X) = \sum_{x=0}^{\infty} x \frac{e^{-l} l^x}{x!} = \sum_{x=1}^{\infty} x \frac{e^{-l} l^x}{x!} = l \sum_{x=1}^{\infty} x \frac{e^{-l} l^x}{x!} = l \sum_{y=0}^{\infty} x \frac{e^{-l} l^y}{y!} = l$$

87.

a. For a one-quarter acre plot, the parameter is (80)(.25) = 20,
so $P(X \le 16) = F(16;20) = .221$

b. The expected number of trees is $\lambda \cdot$(area) = 80(85,000) = 6,800,000.

c. The area of the circle is $\pi r^2 = .031416$ sq. miles or 20.106 acres. Thus X has a Poisson distribution with parameter 20.106

88.

a. P(X = 10 and no violations) = P(no violations | X = 10) · P(X = 10)
= $(.5)^{10}$ · [F(10;10) – F(9;10)]
= (.000977)(.125) = .000122

b. P(y arrive and exactly 10 have no violations)
= P(exactly 10 have no violations | y arrive) · P(y arrive)

= P(10 successes in y trials when p = .5) · $e^{-10}\dfrac{(10)^y}{y!}$

$$= \binom{y}{10}(.5)^{10}(.5)^{y-10}e^{-10}\frac{(10)^y}{y!} = \frac{e^{-10}(5)^y}{10!(y-10)!}$$

c. P(exactly 10 without a violation) = $\displaystyle\sum_{y=10}^{\infty}\frac{e^{-10}(5)^y}{10!(y-10)!}$

$$= \frac{e^{-10}\cdot 5^{10}}{10!}\sum_{y=10}^{\infty}\frac{(5)^{y-10}}{(y-10)!} = \frac{e^{-10}\cdot 5^{10}}{10!}\sum_{u=0}^{\infty}\frac{(5)^u}{(u)!} = \frac{e^{-10}\cdot 5^{10}}{10!}\cdot e^5$$

$$= \frac{e^{-5}\cdot 5^{10}}{10!} = \text{p(10;5)}.$$

In fact, generalizing this argument shows that the number of "no-violation" arrivals within the hour has a Poisson distribution with parameter 5; the 5 results from λp = 10(.5).

89.

a. No events in (0, t+Δt) if and only if no events in (o, t) and no events in (t, t+Δt). Thus, P_0 (t+Δt) = P_0(t) ·P(no events in (t, t+Δt))
= P_0(t)[1 - $\lambda \cdot \Delta$t – o(Δt)]

b. $$\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t} = -l P_0(t)\frac{\Delta' t}{\Delta' t} - P_0(t)\cdot\frac{o(\Delta t)}{\Delta t}$$

c. $\dfrac{d}{dt}\left[e^{-lt}\right]$ = $-\lambda e^{-\lambda t}$ = $-\lambda P_0(t)$, as desired.

d. $$\frac{d}{dt}\left[\frac{e^{-lt}(lt)^k}{k!}\right] = \frac{-l e^{-lt}(lt)^k}{k!} + \frac{k l e^{-lt}(lt)^{k-1}}{k!}$$

$= -l\dfrac{e^{-lt}(lt)^k}{k!} + l\dfrac{e^{-lt}(lt)^{k-1}}{(k-1)!} =$ $-\lambda P_k(t) + \lambda P_{k-1}(t)$ as desired.

Supplementary Exercises

90. Outcomes are(1,2,3)(1,2,4) (1,2,5) … (5,6,7); there are 35 such outcomes. Each having probability $\frac{1}{35}$. The W values for these outcomes are 6 (=1+2+3), 7, 8, …, 18. Since there is just one outcome with W value 6, p(6) = P(W = 6) = $\frac{1}{35}$. Similarly, there are three outcomes with W value 9 [(1,2,6) (1,3,5) and 2,3,4)], so p(9) = $\frac{3}{35}$. Continuing in this manner yields the following distribution:

W	6	7	8	9	10	11	12	13	14	15	16	17	18
P(W)	$\frac{1}{35}$	$\frac{1}{35}$	$\frac{2}{35}$	$\frac{3}{35}$	$\frac{4}{35}$	$\frac{4}{35}$	$\frac{5}{35}$	$\frac{4}{35}$	$\frac{4}{35}$	$\frac{3}{35}$	$\frac{2}{35}$	$\frac{1}{35}$	$\frac{1}{35}$

Since the distribution is symmetric about 12, μ = 12, and $s^2 = \sum_{w=6}^{18}(w-12)^2\,p(w)$

= $\frac{1}{35}$ $[(6)^2(1) + (5)^2(1) + \ldots + (5)^2(1) + (6)^2(1) = 8$

91.

a. p(1) = P(exactly one suit) = P(all spades) + P(all hearts) + P(all diamonds)

+ P(all clubs) = 4P(all spades) = $4\cdot\frac{\binom{13}{5}}{\binom{52}{5}} = .00198$

p(2) = P(all hearts and spades with at least one of each) + …+ P(all diamonds and clubs with at least one of each)
= 6 P(all hearts and spades with at least one of each)
= 6 [P(1 h and 4 s) + P(2 h and 3 s) + P(3 h and 2 s) + P(4 h and 1 s)]

$$=6\cdot\left[2\cdot\frac{\binom{13}{4}\binom{13}{1}}{\binom{52}{5}}+2\cdot\frac{\binom{13}{3}\binom{13}{2}}{\binom{52}{5}}\right]=6\left[\frac{18{,}590+44{,}616}{2{,}598{,}960}\right]=.14592$$

p(4) = 4P(2 spades, 1 h, 1 d, 1 c) = $\frac{4\cdot\binom{13}{2}(13)(13)(13)}{\binom{52}{5}} = .26375$

p(3) = 1 – [p(1) + p(2) + p(4)] = .58835

b. $\mu = \sum_{x=1}^{4} x\cdot p(x) = 3.114,\ s^2 = \left[\sum_{x=1}^{4} x^2\cdot p(x)\right]-(3.114)^2 = .405, s = .636$

92. $p(y) = P(Y = y) = P(y \text{ trials to achieve r S's}) = P(y\text{-r F's before } r^{th} \text{ S})$

$= nb(y - r;r,p) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$, y = r, r+1, r+2, …

93.

a. b(x;15,.75)

b. P(X > 10) = 1 - B(9;15, .75) = 1 - .148

c. B(10;15, .75) - B(5;15, .75) = .314 - .001 = .313

d. $\mu = (15)(.75) = 11.75$, $\sigma^2 = (15)(.75)(.25) = 2.81$

e. Requests can all be met if and only if $X \le 10$, and $15 - X \le 8$, i.e. if $7 \le X \le 10$, so P(all requests met) = B(10; 15,.75) - B(6; 15,.75) = .310

94. P(6-v light works) = P(at least one 6-v battery works) = 1 – P(neither works) $= 1 - (1-p)^2$. P(D light works) = P(at least 2 d batteries work) = 1 – P(at most 1 D battery works) $= 1 - [(1-p)^4 + 4(1-p)^3]$. The 6-v should be taken if $1 - (1-p)^2 \ge 1 - [(1-p)^4 + 4(1-p)^3]$.

Simplifying, $1 \le (1-p)^2 + 4p(1-p) \Rightarrow 0 \le 2p - 3p^3 \Rightarrow p \le \frac{2}{3}$.

95. Let X ~ Bin(5, .9). Then $P(X \ge 3) = 1 - P(X \le 2) = 1 - B(2;5,.9) = .991$

96.

a. $P(X \ge 5) = 1 - B(4;25,.05) = .007$

b. $P(X \ge 5) = 1 - B(4;25,.10) = .098$

c. $P(X \ge 5) = 1 - B(4;25,.20) = .579$

d. All would decrease, which is bad if the % defective is large and good if the % is small.

97.

a. N = 500, p = .005, so np = 2.5 and b(x; 500, .005) $\approx$ p(x; 2.5), a Poisson p.m.f.

b. P(X = 5) = p(5; 2.5) - p(4; 2.5) = .9580 - .8912 = .0668

c. $P(X \ge 5) = 1 - p(4;2.5) = 1 - .8912 = .1088$

98. X ~ B(x; 25, p).

a. B(18; 25, .5) – B(6; 25, .5) = .986

b. B(18; 25, .8) – B(6; 25, .8) = .220

c. With p = .5, P(rejecting the claim) = $P(X \le 7) + P(X \ge 18)$ = .022 + [1 - .978] = .022 + .022 = .044

d. The claim will not be rejected when $8 \le X \le 17$.
With p=.6, $P(8 \le X \le 17)$ = B(17;25,.6) – B(7;25,.6) = .846 - .001 = .845.
With p=.8, $P(8 \le X \le 17)$ = B(17;25,.8) – B(7;25,.8) = .109 - .000 = .109.

e. We want P(rejecting the claim) = .01. Using the decision rule "reject if X = 6 or $X \ge$ 19" gives the probability .014, which is too large. We should use "reject if X = 5 or $X \ge$ 20" which yields P(rejecting the claim) = .002 + .002 = .004.

99. Let Y denote the number of tests carried out. For n = 3, possible Y values are 1 and 4. P(Y = 1) = P(no one has the disease) = $(.9)^3$ = .729 and P(Y = 4) = .271, so E(Y) = (1)(.729) + (4)(.271) = 1.813, as contrasted with the 3 tests necessary without group testing.

100. Regard any particular symbol being received as constituting a trial. Then p = P(S) = P(symbol is sent correctly or is sent incorrectly and subsequently corrected) = $1 - p_1 + p_1p_2$. The block of n symbols gives a binomial experiment with n trials and p = $1 - p_1 + p_1p_2$.

101. p(2) = P(X = 2) = P(S on #1 and S on #2) = p^2
p(3) = P(S on #3 and S on #2 and F on #1) = $(1 – p)p^2$
p(4) = P(S on #4 and S on #3 and F on #2) = $(1 – p)p^2$
p(5) = P(S on #5 and S on #4 and F on #3 and no 2 consecutive S's on trials prior to #3) = [1 – p(2)]$(1 – p)p^2$
p(6) = P(S on #6 and S on #5 and F on #4 and no 2 consecutive S's on trials prior to #4) = [1 – p(2) – p(3)]$(1 – p)p^2$
In general, for x = 5, 6, 7, …: p(x) = [1 – p(2) - … – p(x - 3)]$(1 – p)p^2$
For p = .9,

x	2	3	4	5	6	7	8
p(x)	.81	.081	.081	.0154	.0088	.0023	.0010

So $P(X \le 8)$ = p(2) + … + p(8) = .9995

102.

a. With X ~ Bin(25, .1),$P(2 \le X \le 6)$ = B(6;25,.1 – B(1;25,.1) = .991 - .271 = 720

b. E(X) = np = 25(.1) = 2.5, $\sigma_X = \sqrt{npq} = \sqrt{25(.1)(.9)} = \sqrt{2.25} = 1.50$

c. $P(X \ge 7$ when p = .1) = 1 – B(6;25,.1) = 1 - .991 = .009

d. $P(X \le 6$ when p = .2) = B(6;25,.2) = = .780, which is quite large

103.

a. Let event C = seed carries single spikelets, and event P = seed produces ears with single spikelets. Then P(P ∩ C) = P(P | C) · P(C) = .29 (.40) = .116. Let X = the number of seeds out of the 10 selected that meet the condition P ∩ C. Then X ~ Bin(10, .116).

P(X = 5) = $\binom{10}{5}(.116)^5(.884)^5 = .002857$

b. For 1 seed, the event of interest is P = seed produces ears with single spikelets.
P(P) = P(P ∩ C) + P(P ∩ C′) = .116 (from **a**) + P(P | C′) · P(C′)
= .116 + (.26)(.40) = .272.
Let Y = the number out of the 10 seeds that meet condition P.
Then Y ~ Bin(10, .272), and P(Y = 5) = .0767.
P(Y ≤ 5) = b(0;10,.272) + … + b(5;10,.272) = .041813 + … + .076719 = .97024

104. With S = favored acquittal, the population size is N = 12, the number of population S's is M = 4, the sample size is n = 4, and the p.m.f. of the number of interviewed jurors who favor acquittal is the hypergeometric p.m.f. h(x;4,4,12). E(X) = $4 \cdot \left(\frac{4}{12}\right) = 1.33$

105.

a. P(X = 0) = F(0;2) 0.135

b. Let S = an operator who receives no requests. Then p = .135 and we wish P(4 S's in 5 trials) = b(4;5,..135) = $\binom{5}{4}(.135)^4(.884)^1 = .00144$

c. P(all receive x) = P(first receives x) · … · P(fifth receives x) = $\left[\frac{e^{-2}2^x}{x!}\right]^5$, and P(all receive the same number) is the sum from x = 0 to ∞.

106. P(at least one) = 1 – P(none) = 1 - $e^{-lpR^2} \cdot \frac{(lpR^2)^0}{0!}$ = 1 - e^{-lpR^2} = .99 ⇒ e^{-lpR^2} = .01

$$\Rightarrow R^2 = \frac{-1n(.01)}{lp} = .7329 \Rightarrow R = .8561$$

107. The number sold is min (X, 5), so E[min(x, 5)] = $\sum^{\infty} \min(x,5)\, p(x;4)$

= (0)p(0;4) + (1) p(1;4) + (2) p(2;4) + (3) p(3;4) + (4) p(4;4) + $5\sum_{x=5}^{\infty} p(x;4)$

= 1.735 + 5[1 – F(4;4)] = 3.59

108.

a. P(X = x) = P(A wins in x games) + P(B wins in x games)
= P(9 S's in 1st x-1 ∩ S on the xth) + P(9 F's in 1st x-1 ∩ F on the xth)

$$= \binom{x-1}{9} p^9 (1-p)^{x-10} p + \binom{x-1}{9} (1-p)^9 p^{x-10} (1-p)$$

$$= \binom{x-1}{9} \left[p^{10} (1-p)^{x-10} + (1-p)^{10} p^{x-10} \right]$$

b. Possible values of X are now 10, 11, 12, …(all positive integers ≥ 10). Now

$$P(X = x) = \binom{x-1}{9} \left[p^{10} (1-p)^{x-10} + q^{10} (1-q)^{x-10} \right] \text{ for } x = 10, \dots, 19,$$

So P(X ≥ 20) = 1 – P(X < 20) and P(X < 20) = $\sum_{x=10}^{19} P(X = x)$

109.

a. No; probability of success is not the same for all tests

b. There are four ways exactly three could have positive results. Let D represent those with the disease and D′ represent those without the disease.

Combination		**Probability**
D	D′	
0	3	$\left[\binom{5}{0}(.2)^0(.8)^5\right] \cdot \left[\binom{5}{3}(.9)^3(.1)^2\right]$ =(.32768)(.0729) = .02389
1	2	$\left[\binom{5}{1}(.2)^1(.8)^4\right] \cdot \left[\binom{5}{2}(.9)2(.1)^3\right]$ =(.4096)(.0081) = .00332
2	1	$\left[\binom{5}{2}(.2)^2(.8)^3\right] \cdot \left[\binom{5}{1}(.9)^1(.1)^4\right]$ =(.2048)(.00045) = .00009216
3	0	$\left[\binom{5}{3}(.2)^3(.8)^2\right] \cdot \left[\binom{5}{0}(.9)^0(.1)^5\right]$ =(.0512)(.00001) = .000000512

Adding up the probabilities associated with the four combinations yields 0.0273.

110. k(r,x) = $\dfrac{(x+r-1)(x+r-2)\ldots(x+r-x)}{x!}$

With r = 2.5 and p = .3, p(4) = $\dfrac{(5.5)(4.5)(3.5)(2.5)}{4!}(.3)^{2.5}(.7)^{4} = .1068$

Using k(r,0) = 1, P(X ≥ 1) = 1 – p(0) = 1 – $(.3)^{2.5}$ = .9507

111.

a. p(x; λ,μ) = $\frac{1}{2}p(x;\, l\,) + \frac{1}{2}p(x; m)$ where both p(x;λ) and p(x; μ) are Poisson p.m.f.'s and thus ≥ 0, so p(x; λ,μ) ≥ 0. Further,

$$\sum_{x=0}^{\infty} p(x;\, l\,, m) = \frac{1}{2}\sum_{x=0}^{\infty} p(x;\, l\,) + \frac{1}{2}\sum_{x=0}^{\infty} p(x; m) = \frac{1}{2} + \frac{1}{2} = 1$$

b. $.6\,p(x;\, l\,) + .4\,p(x; m)$

c. E(X) = $\sum_{x=0}^{\infty} x[\frac{1}{2}p(x;\, l\,) + \frac{1}{2}p(x; m)] = \frac{1}{2}\sum_{x=0}^{\infty} xp(x;\, l\,) + \frac{1}{2}\sum_{x=0}^{\infty} xp(x; m)$

= $\dfrac{1}{2}\, l + \dfrac{1}{2}m = \dfrac{l + m}{2}$

d. E(X²) = $\frac{1}{2}\sum_{x=0}^{\infty} x^2 p(x;\, l\,) + \frac{1}{2}\sum_{x=0}^{\infty} x^2 p(x; m) = \frac{1}{2}(l^2 + l\,) + \frac{1}{2}(m^2 + m)$ (since for a Poisson r.v., $E(X^2) = V(X) + [E(X)]^2 = \lambda + \lambda^2$),

so V(X) = $\dfrac{1}{2}\left[l^2 + l + m^2 + m\right] - \left[\dfrac{l + m}{2}\right]^2 = \left(\dfrac{l - m}{2}\right)^2 + \dfrac{l + m}{2}$

112.

a. $\dfrac{b(x+1; n, p)}{b(x; n, p)} = \dfrac{(n-x)}{(x+1)} \cdot \dfrac{p}{(1-p)} > 1$ if np – (1 – p) > x, from which the stated conclusion follows.

b. $\dfrac{p(x+1;\, l\,)}{p(x;\, l\,)} = \dfrac{l}{(x+1)} > 1$ if x < λ - 1 , from which the stated conclusion follows. If λ is an integer, then λ - 1 is a mode, but p(λ,λ) = p(1 - λ, λ) so λ is also a mode[p(x; λ)] achieves its maximum for both x = λ - 1 and x = λ.

113. $P(X = j) = \sum_{i=1}^{10} P$ (arm on track $i \cap X = j$) $= \sum_{i=1}^{10} P$ ($X = j$ | arm on i) $\cdot p_i$

$= \sum_{i=1}^{10} P$ (next seek at I+j+1 or I-j-1) $\cdot p_i$ $= \sum_{i=1}^{10} (p_{i+j+1} + P_{i-j-1}) p_i$

where $p_k = 0$ if $k < 0$ or $k > 10$.

114. $E(X) = \sum_{x=0}^{n} x \cdot \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} = \sum_{x=1}^{n} \frac{\frac{M!}{(x-1)!(M-x)!} \cdot \binom{N-M}{n-x}}{\binom{N}{n}}$

$$n \cdot \frac{M}{N} \sum_{x=1}^{n} \binom{M-1}{x-1} \frac{\binom{N-M}{n-x}}{\binom{N-1}{n-1}} = n \cdot \frac{M}{N} \sum_{y=0}^{n-1} \binom{M-1}{y} \frac{\binom{N-1-(M-1)}{n-1-y}}{\binom{N-1}{n-1}}$$

$$n \cdot \frac{M}{N} \sum_{y=0}^{n-1} h(y; n-1, M-1, N-1) = n \cdot \frac{M}{N}$$

115. Let $A = \{x: |x - \mu| \ge k\sigma\}$. Then $\sigma^2 = \sum_A (x-m)^2 p(x) \ge (ks)^2 \sum_A p(x)$. But $\sum_A p(x)$ = P(X is in A) = $P(|X - \mu| \ge k\sigma)$, so $\sigma^2 \ge k^2\sigma^2 \cdot P(|X - \mu| \ge k\sigma)$, as desired.

116.

a. For [0,4], $\lambda = \int_0^4 e^{2+.6t} dt$ = 123.44, whereas for [2,6], $\lambda = \int_2^6 e^{2+.6t} dt$ = 409.82

b. $\lambda = \int_0^{0.9907} e^{2+.6t} dt$ = 9.9996 ≈ 10, so the desired probability is F(15, 10) = .951.