CHAPTER 2

Section 2.1

1.

a. S = { 1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3124, 3142, 4123, 4132, 3214, 3241, 4213, 4231 }

b. Event A contains the outcomes where 1 is first in the list:
A = { 1324, 1342, 1423, 1432 }

c. Event B contains the outcomes where 2 is first or second:
B = { 2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231 }

d. The compound event $A \cup B$ contains the outcomes in A or B or both:
$A \cup B$ = {1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231 }

2.

a. Event A = { RRR, LLL, SSS }

b. Event B = { RLS, RSL, LRS, LSR, SRL, SLR }

c. Event C = { RRL, RRS, RLR, RSR, LRR, SRR }

d. Event D = { RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS }

e. Event D' contains outcomes where all cars go the same direction, or they all go different directions:
D' = { RRR, LLL, SSS, RLS, RSL, LRS, LSR, SRL, SLR }

Because Event D totally encloses Event C, the compound event $C \cup D = D$:
$C \cup D$ = { RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS }

Using similar reasoning, we see that the compound event $C \cap D = C$:
$C \cap D$ = { RRL, RRS, RLR, RSR, LRR, SRR }

3.

a. Event A = { SSF, SFS, FSS }

b. Event B = { SSS, SSF, SFS, FSS }

c. For Event C, the system must have component 1 working (S in the first position), then at least one of the other two components must work (at least one S in the 2nd and 3rd positions: Event C = { SSS, SSF, SFS }

d. Event C′ = { SFF, FSS, FSF, FFS, FFF }
Event A∪C = { SSS, SSF, SFS, FSS }
Event A∩C = { SSF, SFS }
Event B∪C = { SSS, SSF, SFS, FSS }
Event B∩C = { SSS SSF, SFS }

4.

a.

	Home Mortgage Number			
Outcome	1	2	3	4
1	F	F	F	F
2	F	F	F	V
3	F	F	V	F
4	F	F	V	V
5	F	V	F	F
6	F	V	F	V
7	F	V	V	F
8	F	V	V	V
9	V	F	F	F
10	V	F	F	V
11	V	F	V	F
12	V	F	V	V
13	V	V	F	F
14	V	V	F	V
15	V	V	V	F
16	V	V	V	V

b. Outcome numbers 2, 3, 5 ,9

c. Outcome numbers 1, 16

d. Outcome numbers 1, 2, 3, 5, 9

e. In words, the UNION described is the event that either all of the mortgages are variable, or that at most all of them are variable: outcomes 1,2,3,5,9,16. The INTERSECTION described is the event that all of the mortgages are fixed: outcome 1.

f. The UNION described is the event that either exactly three are fixed, or that all four are the same: outcomes 1, 2, 3, 5, 9, 16. The INTERSECTION in words is the event that exactly three are fixed AND that all four are the same. This cannot happen. (There are no outcomes in common) : **b** ∩ **c** = ∅.

5.

a.

Outcome Number	Outcome
1	111
2	112
3	113
4	121
5	122
6	123
7	131
8	132
9	133
10	211
11	212
12	213
13	221
14	222
15	223
16	231
17	232
18	233
19	311
20	312
21	313
22	321
23	322
24	323
25	331
26	332
27	333

b. Outcome Numbers 1, 14, 27

c. Outcome Numbers 6, 8, 12, 16, 20, 22

d. Outcome Numbers 1, 3, 7, 9, 19, 21, 25, 27

6.

a.

Outcome Number	Outcome
1	123
2	124
3	125
4	213
5	214
6	215
7	13
8	14
9	15
10	23
11	24
12	25
13	3
14	4
15	5

b. Outcomes 13, 14, 15

c. Outcomes 3, 6, 9, 12, 15

d. Outcomes 10, 11, 12, 13, 14, 15

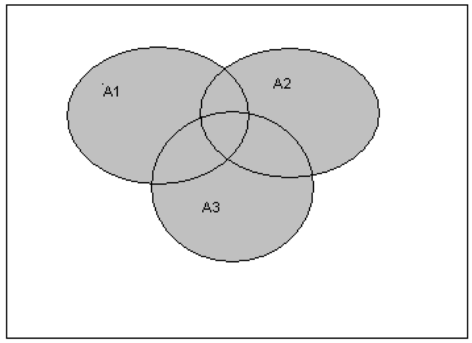
7.

a. S = {BBBAAAA, BBABAAA, BBAABAA, BBAAABA, BBAAAAB, BABBAAA, BABABAA, BABAABA, BABAAAB, BAABBAA, BAABABA, BAABAAB, BAAABBA, BAAABAB, BAAAABB, ABBBAAA, ABBABAA, ABBAABA, ABBAAAB, ABABBAA, ABABABA, ABABAAB, ABAABBA, ABAABAB, ABAAABB, AABBBAA, AABBABA, AABBAAB, AABABBA, AABABAB, AABAABB, AAABBBA, AAABBAB, AAABABB, AAAABBB}

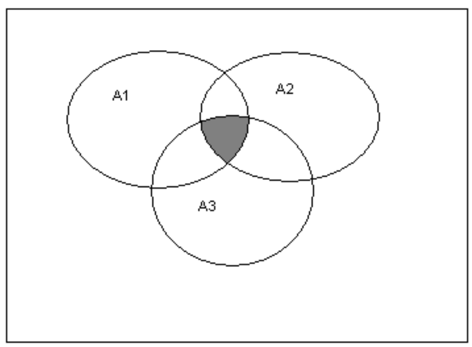
b. {AAAABBB, AAABABB, AAABBAB, AABAABB, AABABAB}

8.

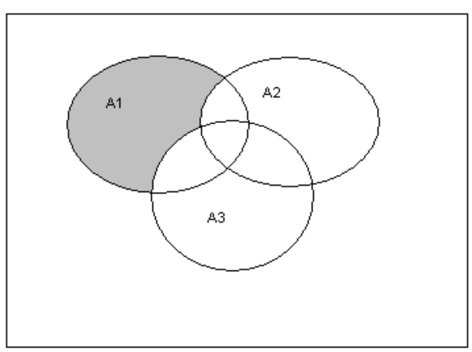
a. $A_1 \cup A_2 \cup A_3$



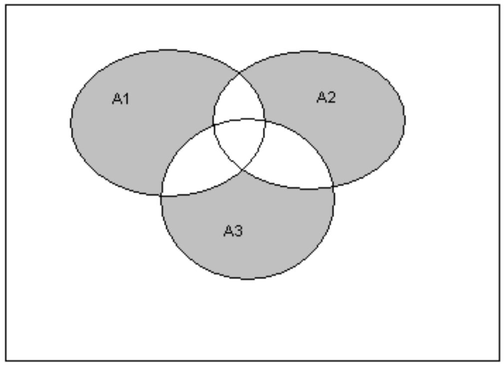
b. $A_1 \cap A_2 \cap A_3$



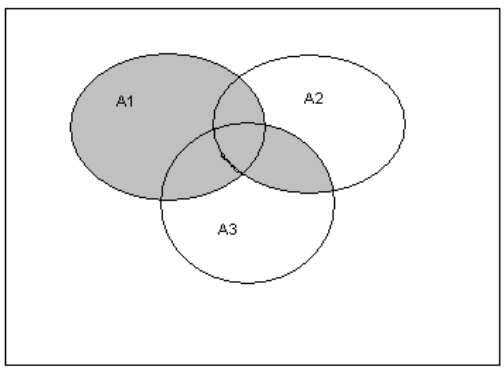
c. $A_1 \cap A_2' \cap A_3'$



d. $(A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)$

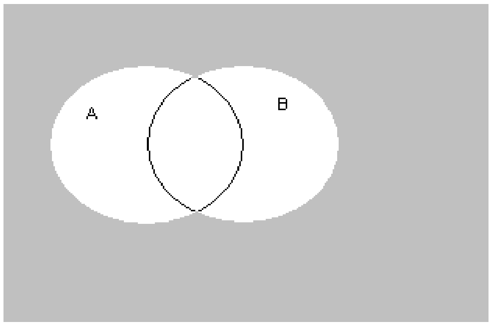


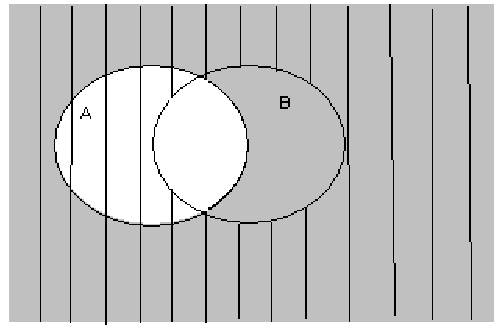
e. $A_1 \cup (A_2 \cap A_3)$



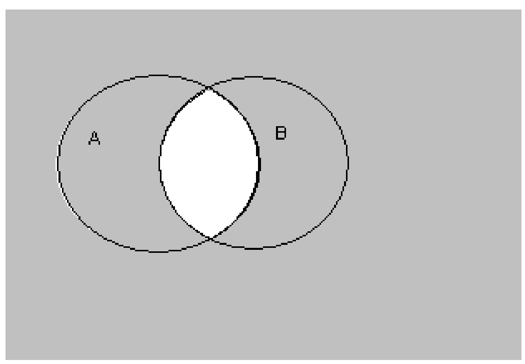
9.

a. In the diagram on the left, the shaded area is $(A \cup B)'$. On the right, the shaded area is A', the striped area is B', and the intersection $A' \cap B'$ occurs where there is BOTH shading and stripes. These two diagrams display the same area.





b. In the diagram below, the shaded area represents $(A \cap B)'$. Using the diagram on the right above, the union of A' and B' is represented by the areas that have either shading or stripes or both. Both of the diagrams display the same area.



10.

a. A = {Chev, Pont, Buick}, B = {Ford, Merc}, C = {Plym, Chrys} are three mutually exclusive events.

b. No, let E = {Chev, Pont}, F = {Pont, Buick}, G = {Buick, Ford}. These events are not mutually exclusive (e.g. E and F have an outcome in common), yet there is no outcome common to all three events.

Section 2.2

11.

a. .07

b. .15 + .10 + .05 = .30

c. Let event A = selected customer owns stocks. Then the probability that a selected customer does not own a stock can be represented by
$P(A') = 1 - P(A) = 1 – (.18 + .25) = 1 - .43 = .57$. This could also have been done easily by adding the probabilities of the funds that are not stocks.

12.

a. $P(A \cup B) = .50 + .40 - .25 = .65$

b. $P(A \cup B)' = 1 - .65 = .35$

c. $A \cap B'$; $P(A \cap B') = P(A) – P(A \cap B) = .50 - .25 = .25$

13.

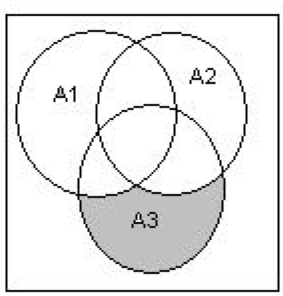
a. awarded either #1 or #2 (or both):
$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = .22 + .25 - .11 = .36$

b. awarded neither #1 or #2:
$P(A_1' \cap A_2') = P[(A_1 \cup A_2)'] = 1 - P(A_1 \cup A_2) = 1 - .36 = .64$

c. awarded at least one of #1, #2, #3:
$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) – P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$
$$= .22 +.25 + .28 - .11 -.05 - .07 + .01 = .53$$

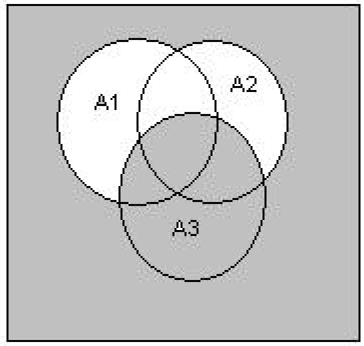
d. awarded none of the three projects:
$P(A_1' \cap A_2' \cap A_3') = 1 – P(\text{awarded at least one}) = 1 - .53 = .47.$

e. awarded #3 but neither #1 nor #2:
$$P(A_1' \cap A_2' \cap A_3) = P(A_3) - P(A_1 \cap A_3) – P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$
$$= .28 - .05 - .07+ .01 \quad = .17$$

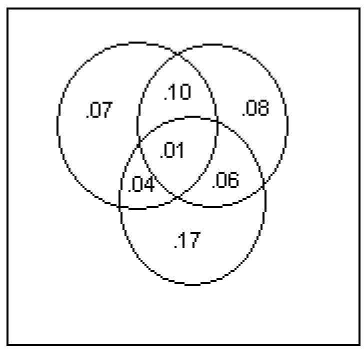


f. either (neither #1 nor #2) or #3:

$$P[(A_1' \cap A_2') \cup A_3] = P(\text{shaded region}) = P(\text{awarded none}) + P(A_3)$$
$$= .47 + .28 = .75$$



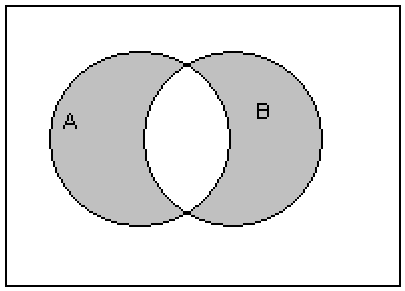
Alternatively, answers to **a – f** can be obtained from probabilities on the accompanying Venn diagram



14.

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$,
so $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= .8 + .7 - .9 = .6$

b. $P(\text{shaded region}) = P(A \cup B) - P(A \cap B) = .9 - .6 = .3$
Shaded region = event of interest = $(A \cap B') \cup (A' \cap B)$



15.

a. Let event E be the event that at most one purchases an electric dryer. Then E′ is the event that at least two purchase electric dryers.
$P(E') = 1 - P(E) = 1 - .428 = .572$

b. Let event A be the event that all five purchase gas. Let event B be the event that all five purchase electric. All other possible outcomes are those in which at least one of each type is purchased. Thus, the desired probability =
$1 - P(A) - P(B) = 1 - .116 - .005 = .879$

16.

a. There are six simple events, corresponding to the outcomes CDP, CPD, DCP, DPC, PCD, and PDC. The probability assigned to each is $\frac{1}{6}$.

b. P(C ranked first) = P({CPD, CDP}) = $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = .333$

c. P(C ranked first and D last) = P({CPD}) = $\frac{1}{6}$

17.

a. The probabilities do not add to 1 because there are other software packages besides SPSS and SAS for which requests could be made.

b. $P(A') = 1 - P(A) = 1 - .30 = .70$

c. $P(A \cup B) = P(A) + P(B) = .30 + .50 = .80$
(since A and B are mutually exclusive events)

d. $P(A' \cap B') = P[(A \cup B)']$ (De Morgan's law)
$= 1 - P(A \cup B)$
$=1 - .80 = .20$

18. This situation requires the complement concept. The only way for the desired event NOT to happen is if a 75 W bulb is selected first. Let event A be that a 75 W bulb is selected first, and $P(A) = \frac{6}{15}$. Then the desired event is event A′.
So $P(A') = 1 - P(A) = 1 - \frac{6}{15} = \frac{9}{15} = .60$

19. Let event A be that the selected joint was found defective by inspector A. $P(A) = \frac{724}{10{,}000}$. Let event B be analogous for inspector B. $P(B) = \frac{751}{10{,}000}$. Compound event $A \cup B$ is the event that the selected joint was found defective by at least one of the two inspectors. $P(A \cup B) = \frac{1159}{10{,}000}$.

a. The desired event is $(A \cup B)'$, so we use the complement rule:
$P(A \cup B)' = 1 - P(A \cup B) = 1 - \frac{1159}{10{,}000} = \frac{8841}{10{,}000} = .8841$

b. The desired event is $B \cap A'$. $P(B \cap A') = P(B) - P(A \cap B)$.
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$,
$= .0724 + .0751 - .1159 = .0316$
So $P(B \cap A') = P(B) - P(A \cap B)$
$= .0751 - .0316 = .0435$

20. Let S1, S2 and S3 represent the swing and night shifts, respectively. Let C1 and C2 represent the unsafe conditions and unrelated to conditions, respectively.

a. The simple events are {S1,C1}, {S1,C2}, {S2,C1}, {S2,C2},{S3,C1}, {S3,C2}.

b. P({C1})= P({S1,C1},{S2,C1},{S3,C1})= .10 + .08 + .05 = .23

c. P({S1}′) = 1 - P({S1,C1}, {S1,C2}) = 1 – (.10 + .35) = .55

21.

a. $P(\{M,H\}) = .10$

b. P(low auto) = P[{(L,N}, (L,L), (L,M), (L,H)}] = .04 + .06 + .05 + .03 = .18 Following a similar pattern, P(low homeowner's) = .06 + .10 + .03 = .19

c. P(same deductible for both) = P[{ LL, MM, HH }] = .06 + .20 + .15 = .41

d. P(deductibles are different) = 1 – P(same deductibles) = 1 - .41 = .59

e. P(at least one low deductible) = P[{ LN, LL, LM, LH, ML, HL }]
= .04 + .06 + .05 + .03 + .10 + .03 = .31

f. P(neither low) = 1 – P(at least one low) = 1 - .31 = .69

22.

a. $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = .4 + .5 - .6 = .3$

b. $P(A_1 \cap A_2') = P(A_1) - P(A_1 \cap A_2) = .4 - .3 = .1$

c. P(exactly one) $= P(A_1 \cup A_2) - P(A_1 \cap A_2) = .6 - .3 = .3$

23. Assume that the computers are numbered 1 – 6 as described. Also assume that computers 1 and 2 are the laptops. Possible outcomes are (1,2) (1,3) (1,4) (1,5) (1,6) (2,3) (2,4) (2,5) (2,6) (3,4) (3,5) (3,6) (4,5) (4,6) and (5,6).

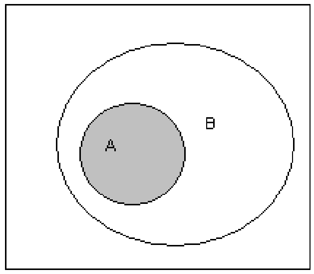
a. P(both are laptops) = P[{ (1,2)}] = $\frac{1}{15}$ =.067

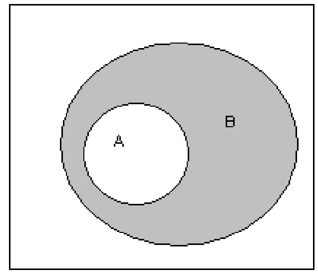
b. P(both are desktops) = P[{(3,4) (3,5) (3,6) (4,5) (4,6) (5,6)}] = $\frac{6}{15}$ = .40

c. P(at least one desktop) = 1 – P(no desktops)
= 1 – P(both are laptops)
= 1 – .067 = .933

d. P(at least one of each type) = 1 – P(both are the same)
= 1 – P(both laptops) – P(both desktops)
= 1 - .067 - .40 = .533

24. Since A is contained in B, then B can be written as the union of A and $(B \cap A')$, two mutually exclusive events. (See diagram).



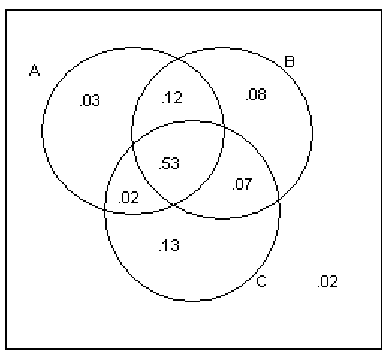


From Axiom 3, $P[A \cup (B \cap A')] = P(A) + P(B \cap A')$. Substituting P(B), $P(B) = P(A) + P(B \cap A')$ or $P(B) - P(A) = P(B \cap A')$. From Axiom 1, $P(B \cap A') \geq 0$, so $P(B) \geq P(A)$ or $P(A) \leq P(B)$. For general events A and B, $P(A \cap B) \leq P(A)$, and $P(A \cup B) \geq P(A)$.

25. $P(A \cap B) = P(A) + P(B) - P(A \cup B) = .65$

$P(A \cap C) = .55$, $P(B \cap C) = .60$

$$\begin{aligned} P(A \cap B \cap C) &= P(A \cup B \cup C) - P(A) - P(B) - P(C) \\ &\quad + P(A \cap B) + P(A \cap C) + P(B \cap C) \\ &= .98 - .7 - .8 - .75 + .65 + .55 + .60 \\ &= .53 \end{aligned}$$



a. $P(A \cup B \cup C) = .98$, as given.

b. P(none selected) $= 1 - P(A \cup B \cup C) = 1 - .98 = .02$

c. P(only automatic transmission selected) = .03 from the Venn Diagram

d. P(exactly one of the three) = .03 + .08 + .13 = .24

26.

a. $P(A_1') = 1 - P(A_1) = 1 - .12 = .88$

b. $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = .12 + .07 - .13 = .06$

c. $P(A_1 \cap A_2 \cap A_3') = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) = .06 - .01 = .05$

d. P(at most two errors) = 1 – P(all three types)
$= 1 - P(A_1 \cap A_2 \cap A_3)$
$= 1 - .01 = .99$

27. Outcomes: $(A,B)\ (A,C_1)\ (A,C_2)\ (A,F)\ (B,A)\ (B,C_1)\ (B,C_2)\ (B,F)$
$(C_1,A)\ (C_1,B)\ (C_1,C_2)\ (C_1,F)\ (C_2,A)\ (C_2,B)\ (C_2,C_1)\ (C_2,F)$
$(F,A)\ (F,B)\ (F,C_1)\ (F,C_2)$

a. P[(A,B) or (B,A)] = $\frac{2}{20} = \frac{1}{10} = .1$

b. P(at least one C) = $\frac{14}{20} = \frac{7}{10} = .7$

c. P(at least 15 years) = 1 – P(at most 14 years)
= 1 – P[(3,6) or (6,3) or (3,7) or (7,3) or (3,10) or (10,3) or (6,7) or (7,6)]
$= 1 - \frac{8}{20} = 1 - .4 = .6$

28. There are 27 equally likely outcomes.

a. P(all the same) = P[(1,1,1) or (2,2,2) or (3,3,3)] = $\frac{3}{27} = \frac{1}{9}$

b. P(at most 2 are assigned to the same station) = 1 – P(all 3 are the same)
$= 1 - \frac{3}{27} = \frac{24}{27} = \frac{8}{9}$

c. P(all different) = [{(1,2,3) (1,3,2) (2,1,3) (2,3,1) (3,1,2) (3,2,1)}]
$= \frac{6}{27} = \frac{2}{9}$

Section 2.3

29.

a. (5)(4) = 20 (5 choices for president, 4 remain for vice president)

b. (5)(4)(3) = 60

c. $\binom{5}{2} = \frac{5!}{2!3!} = 10$ (No ordering is implied in the choice)

30.

a. Because order is important, we'll use $P_{8,3} = 8(7)(6) = 336$.

b. Order doesn't matter here, so we use $C_{30,6} = 593{,}775$.

c. From each group we choose 2: $\binom{8}{2} \bullet \binom{10}{2} \bullet \binom{12}{2} = 83{,}160$

d. The numerator comes from part c and the denominator from part b: $\frac{83{,}160}{593{,}775} = .14$

e. We use the same denominator as in part d. We can have all zinfandel, all merlot, or all cabernet, so P(all same) = P(all z) + P(all m) + P(all c) =

$$\frac{\binom{8}{6} + \binom{10}{6} + \binom{12}{6}}{\binom{30}{6}} = \frac{1162}{593{,}775} = .002$$

31.

a. $(n_1)(n_2) = (9)(27) = 243$

b. $(n_1)(n_2)(n_3) = (9)(27)(15) = 3645$, so such a policy could be carried out for 3645 successive nights, or approximately 10 years, without repeating exactly the same program.

32.

a. $5\times4\times3\times4 = 240$

b. $1\times1\times3\times4 = 12$

c. $4\times3\times3\times3 = 108$

d. # with at least on Sony = total # - # with no Sony = 240 – 108 = 132

e. P(at least one Sony) = $\frac{132}{240} = .55$

P(exactly one Sony) = P(only Sony is receiver)
+ P(only Sony is CD player)
+ P(only Sony is deck)

$$= \frac{1\times3\times3\times3}{240} + \frac{4\times1\times3\times3}{240} + \frac{4\times3\times3\times1}{240} = \frac{27+36+36}{240}$$

$$= \frac{99}{240} = .413$$

33.

a. $\binom{25}{5} = \frac{25!}{5!20!} = 53{,}130$

b. $\binom{8}{4} \bullet \binom{17}{1} = 1190$

c. P(exactly 4 have cracks) = $\frac{\binom{8}{4}\binom{17}{1}}{\binom{25}{5}} = \frac{1190}{53{,}130} = .022$

d. P(at least 4) = P(exactly 4) + P(exactly 5)

$$= \frac{\binom{8}{4}\binom{17}{1}}{\binom{25}{5}} + \frac{\binom{8}{5}\binom{17}{0}}{\binom{25}{5}} = .022 + .001 = .023$$

34.

a. $\binom{20}{6} = 38{,}760$. P(all from day shift) = $\dfrac{\binom{20}{6}\binom{25}{0}}{\binom{45}{6}} = \dfrac{38{,}760}{8{,}145{,}060} = .0048$

b. P(all from same shift) = $\dfrac{\binom{20}{6}\binom{25}{0}}{\binom{45}{6}} + \dfrac{\binom{15}{6}\binom{30}{0}}{\binom{45}{6}} + \dfrac{\binom{10}{6}\binom{35}{0}}{\binom{45}{6}}$

= .0048 + .0006 + .0000 = .0054

c. P(at least two shifts represented) = 1 – P(all from same shift)
= 1 - .0054 = .9946

d. Let A_1 = day shift unrepresented, A_2 = swing shift unrepresented, and A_3 = graveyard shift unrepresented. Then we wish $P(A_1 \cup A_2 \cup A_3)$.
$P(A_1)$ = P(day unrepresented) = P(all from swing and graveyard)

$P(A_1) = \dfrac{\binom{25}{6}}{\binom{45}{6}}$, $P(A_2) = \dfrac{\binom{30}{6}}{\binom{45}{6}}$, $P(A_3) = \dfrac{\binom{35}{6}}{\binom{45}{6}}$,

$P(A_1 \cap A_2)$ = P(all from graveyard) = $\dfrac{\binom{10}{6}}{\binom{45}{6}}$

$P(A_1 \cap A_3) = \dfrac{\binom{15}{6}}{\binom{45}{6}}$, $P(A_2 \cap A_3) = \dfrac{\binom{20}{6}}{\binom{45}{6}}$, $P(A_1 \cap A_2 \cap A_3) = 0$,

So $P(A_1 \cup A_2 \cup A_3) = \dfrac{\binom{25}{6}}{\binom{45}{6}} + \dfrac{\binom{30}{6}}{\binom{45}{6}} + \dfrac{\binom{35}{6}}{\binom{45}{6}} - \dfrac{\binom{10}{6}}{\binom{45}{6}} - \dfrac{\binom{15}{6}}{\binom{45}{6}} - \dfrac{\binom{20}{6}}{\binom{45}{6}}$

= .2939 - .0054 = .2885

35. There are 10 possible outcomes -- $\binom{5}{2}$ ways to select the positions for B's votes: BBAAA, BABAA, BAABA, BAAAB, ABBAA, ABABA, ABAAB, AABBA, AABAB, and AAABB. Only the last two have A ahead of B throughout the vote count. Since the outcomes are equally likely, the desired probability is $\frac{2}{10} = .20$.

36.

a. $n_1 = 3$, $n_2 = 4$, $n_3 = 5$, so $n_1 \times n_2 \times n_3 = 60$ runs

b. $n_1 = 1$, (just one temperature), $n_2 = 2$, $n_3 = 5$ implies that there are 10 such runs.

37. There are $\binom{60}{5}$ ways to select the 5 runs. Each catalyst is used in 12 different runs, so the number of ways of selecting one run from each of these 5 groups is 12^5. Thus the desired probability is $\frac{12^5}{\binom{60}{5}} = .0456$.

38.

a. P(selecting 2 - 75 watt bulbs) = $\frac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \frac{15 \cdot 9}{455} = .2967$

b. P(all three are the same) = $\frac{\binom{4}{3}+\binom{5}{3}+\binom{6}{3}}{\binom{15}{3}} = \frac{4+10+20}{455} = .0747$

c. $\binom{4}{1}\binom{5}{1}\binom{6}{1} = \frac{120}{455} = .2637$

d. To examine exactly one, a 75 watt bulb must be chosen first. (6 ways to accomplish this). To examine exactly two, we must choose another wattage first, then a 75 watt. (9×6 ways). Following the pattern, for exactly three, $9 \times 8 \times 6$ ways; for four, $9 \times 8 \times 7 \times 6$; for five, $9 \times 8 \times 7 \times 6 \times 6$.

P(examine at least 6 bulbs) = 1 – P(examine 5 or less)
= 1 – P(examine exactly 1 or 2 or 3 or 4 or 5)
= 1 – [P(one) + P(two) + … + P(five)]

$$= 1 - \left[\frac{6}{15} + \frac{9\times6}{15\times14} + \frac{9\times8\times6}{15\times14\times13} + \frac{9\times8\times7\times6}{15\times14\times13\times12} + \frac{9\times8\times7\times6\times6}{15\times14\times13\times12\times11}\right]$$

= 1 – [.4 + .2571 + .1582 + .0923 + .0503]
= 1 - .9579 = .0421

39.

a. We want to choose all of the 5 cordless, and 5 of the 10 others, to be among the first 10 serviced, so the desired probability is $\dfrac{\binom{5}{5}\binom{10}{5}}{\binom{15}{10}} = \dfrac{252}{3003} = .0839$

b. Isolating one group, say the cordless phones, we want the other two groups represented in the last 5 serviced. So we choose 5 of the 10 others, except that we don't want to include the outcomes where the last five are all the same.

So we have $\dfrac{\binom{10}{5} - 2}{\binom{15}{5}}$. But we have three groups of phones, so the desired probability is

$$\frac{3\cdot\left[\binom{10}{5} - 2\right]}{\binom{15}{5}} = \frac{3(250)}{3003} = .2498 .$$

c. We want to choose 2 of the 5 cordless, 2 of the 5 cellular, and 2 of the corded phones:

$$\frac{\binom{5}{2}\binom{5}{2}\binom{5}{2}}{\binom{15}{6}} = \frac{1000}{5005} = .1998$$

40.

a. If the A's are distinguishable from one another, and similarly for the B's, C's and D's, then there are 12! Possible chain molecules. Six of these are:

$A_1A_2A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$, $A_1A_3A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$
$A_2A_1A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$, $A_2A_3A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$
$A_3A_1A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$, $A_3A_2A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$

These 6 (=3!) differ only with respect to ordering of the 3 A's. In general, groups of 6 chain molecules can be created such that within each group only the ordering of the A's is different. When the A subscripts are suppressed, each group of 6 "collapses" into a single molecule (B's, C's and D's are still distinguishable). At this point there are $\frac{12!}{3!}$ molecules. Now suppressing subscripts on the B's, C's and D's in turn gives ultimately $\frac{12!}{(3!)^4} = 369{,}600$ chain molecules.

b. Think of the group of 3 A's as a single entity, and similarly for the B's, C's, and D's. Then there are 4! Ways to order these entities, and thus 4! Molecules in which the A's are contiguous, the B's, C's, and D's are also. Thus, P(all together) = $\frac{4!}{369{,}600} = .00006494$.

41.

a. P(at least one F among 1^{st} 3) = 1 – P(no F's among 1^{st} 3)

$$= 1 - \frac{4 \times 3 \times 2}{8 \times 7 \times 6} = 1 - \frac{24}{336} = 1 - .0714 = .9286$$

An alternative method to calculate P(no F's among 1^{st} 3) would be to choose none of the females and 3 of the 4 males, as follows:

$$\frac{\binom{4}{0}\binom{4}{3}}{\binom{8}{3}} = \frac{4}{56} = .0714$$, obviously producing the same result.

b. P(all F's among 1^{st} 5) = $\frac{\binom{4}{4}\binom{4}{1}}{\binom{8}{5}} = \frac{4}{56} = .0714$

c. P(orderings are different) = 1 – P(orderings are the same for both semesters)
= 1 – (# orderings such that the orders are the same each semester)/(total # of possible orderings for 2 semesters)

$$= 1 - \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times) \times (8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)} = .99997520$$

42. Seats:

1	2	3	4	5	6

P(J&P in 1&2) $= \dfrac{2\times1\times4\times3\times2\times1}{6\times5\times4\times3\times2\times1} = \dfrac{1}{15} = .0667$

P(J&P next to each other) = P(J&P in 1&2) + … + P(J&P in 5&6)

$= 5\times\dfrac{1}{15} = \dfrac{1}{3} = .333$

P(at least one H next to his W) = 1 – P(no H next to his W)
We count the # of ways of no H next to his W as follows:
if orderings without a H-W pair in seats #1 and 3 and no H next to his W = $6^{*} \times 4 \times 1^{*} \times 2^{\#} \times 1 \times 1 = 48$
*= pair, # =can't put the mate of seat #2 here or else a H-W pair would be in #5 and 6.

# of orderings without a H-W pair in seats #1 and 3, and no H next to his W = $6 \times 4 \times 2^{\#} \times 2 \times 2 \times 1 = 192$
#= can't be mate of person in seat #1 or #2.
So, # of seating arrangements with no H next to W = 48 + 192 = 240

And P(no H next to his W) = $= \dfrac{240}{6\times5\times4\times3\times2\times1} = \dfrac{1}{3}$, so

P(at least one H next to his W) = 1 - $\dfrac{1}{3} = \dfrac{2}{3}$

43. # of 10 high straights = 4×4×4×4×4 (4 – 10's, 4 – 9's , etc)

P(10 high straight) = $\dfrac{4^5}{\binom{52}{5}} = \dfrac{1024}{2{,}598{,}960} = .000394$

P(straight) = $10\times\dfrac{4^5}{\binom{52}{5}} = .003940$ (Multiply by 10 because there are 10 different card values that could be high: Ace, King, etc.) There are only 40 straight flushes (10 in each suit), so

P(straight flush) = $\dfrac{40}{\binom{52}{5}} = .00001539$

44. $\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n!}{(n-k)!k!} = \binom{n}{n-k}$

The number of subsets of size k = the number of subsets of size n-k, because to each subset of size k there corresponds exactly one subset of size n-k (the n-k objects not in the subset of size k).

Section 2.4

45.

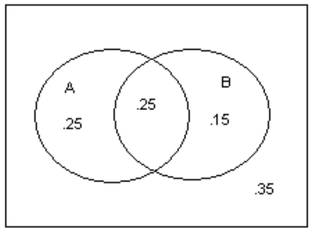
a. P(A) = .106 + .141 + .200 = .447, P(C) =.215 + .200 + .065 + .020 = .500 P(A $\cap$ C) = .200

b. P(A|C) = $\frac{P(A\cap C)}{P(C)} = \frac{.200}{.500} = .400$. If we know that the individual came from ethnic group 3, the probability that he has type A blood is .40. P(C|A) = $\frac{P(A\cap C)}{P(A)} = \frac{.200}{.447} = .447$. If a person has type A blood, the probability that he is from ethnic group 3 is .447

c. Define event D = {ethnic group 1 selected}. We are asked for P(D|B′) = $\frac{P(D\cap B')}{P(B')} = \frac{.200}{.500} = .400$. P(D$\cap$B′)=.082 + .106 + .004 = .192, P(B′) = 1 – P(B) = 1 – [.008 + .018 + .065] = .909

46. Let event A be that the individual is more than 6 feet tall. Let event B be that the individual is a professional basketball player. Then P (A | B) = the probability of the individual being more than 6 feet tall, knowing that the individual is a professional basketball player, and P (B | A) = the probability of the individual being a professional basketball player, knowing that the individual is more than 6 feet tall. P (A | B) will be larger. Most professional BB players are tall, so the probability of an individual in that reduced sample space being more than 6 feet tall is very large. The number of individuals that are pro BB players is small in relation to the # of males more than 6 feet tall.

47.



a. P(B | A) = $\dfrac{P(A \cap B)}{P(A)} = \dfrac{.25}{.50} = .50$

b. P(B′ | A) = $\dfrac{P(A \cap B')}{P(A)} = \dfrac{.25}{.50} = .50$

c. P(A | B) = $\dfrac{P(A \cap B)}{P(B)} = \dfrac{.25}{.40} = .6125$

d. P(A′ | B) = $\dfrac{P(A' \cap B)}{P(B)} = \dfrac{.15}{.40} = .3875$

e. P(A | A∪B) = $\dfrac{P[A \cap (A \cup B)]}{P(A \cup B)} = \dfrac{.50}{.65} = .7692$

48.

a. $P(A_2 \mid A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} = \dfrac{.06}{.12} = .50$

b. $P(A_1 \cap A_2 \cap A_3 \mid A_1) = \dfrac{.01}{.12} = .0833$

c. We want P[(exactly one) | (at least one)].

P(at least one) $= P(A_1 \cup A_2 \cup A_3)$

$= .12 + .07 + .05 - .06 - .03 - .02 + .01 = .14$

Also notice that the intersection of the two events is just the 1st event, since "exactly one" is totally contained in "at least one."

So P[(exactly one) | (at least one)]= $\dfrac{.04 + .01}{.14} = .3571$

d. The pieces of this equation can be found in your answers to exercise 26 (section 2.2):

$$P(A_3' \mid A_1 \cap A_2) = \frac{P(A_1 \cap A_2 \cap A_3')}{P(A_1 \cap A_2)} = \frac{.05}{.06} = .833$$

49. The first desired probability is P(both bulbs are 75 watt $|$ at least one is 75 watt).
P(at least one is 75 watt) = 1 – P(none are 75 watt)

$$= 1 - \frac{\binom{9}{2}}{\binom{15}{2}} = 1 - \frac{36}{105} = \frac{69}{105}.$$

Notice that P[(both are 75 watt)$\cap$(at least one is 75 watt)]

$$= \text{P(both are 75 watt)} = \frac{\binom{6}{2}}{\binom{15}{2}} = \frac{15}{105}.$$

So P(both bulbs are 75 watt $|$ at least one is 75 watt) = $\dfrac{\frac{15}{105}}{\frac{69}{105}} = \dfrac{15}{69} = .2174$

Second, we want P(same rating $|$ at least one NOT 75 watt).
P(at least one NOT 75 watt) = 1 – P(both are 75 watt)

$$= 1 - \frac{15}{105} = \frac{90}{105}.$$

Now, P[(same rating)$\cap$(at least one not 75 watt)] = P(both 40 watt or both 60 watt).

$$\text{P(both 40 watt or both 60 watt)} = \frac{\binom{4}{2} + \binom{5}{2}}{\binom{15}{2}} = \frac{16}{105}$$

Now, the desired conditional probability is $\dfrac{\frac{16}{105}}{\frac{90}{105}} = \dfrac{16}{90} = .1778$

50.

a. P(M $\cap$ LS $\cap$ PR) = .05, directly from the table of probabilities

b. P(M $\cap$ Pr) = P(M,Pr,LS) + P(M,Pr,SS) = .05+.07=.12

c. P(SS) = sum of 9 probabilities in SS table = 56, P(LS) = 1 = .56 = .44

d. P(M) = .08+.07+.12+.10+.05+.07 = .49
P(Pr) = .02+.07+.07+.02+.05+.02 = .25

e. P(M|SS ∩ Pl) = $\dfrac{P(M \cap SS \cap Pl)}{P(SS \cap Pl)} = \dfrac{.08}{.04 + .08 + .03} = .533$

f. P(SS|M ∩ Pl) = $\dfrac{P(SS \cap M \cap Pl)}{P(M \cap Pl)} = \dfrac{.08}{.08 + .10} = .444$

P(LS|M Pl) = 1 - P(SS|M Pl) = 1 - .444 = .556

51.

a. P(R from 1st ∩ R from 2nd) = P(R from 2nd | R from 1st) • P(R from 1st)

$$= \frac{8}{11} \bullet \frac{6}{10} = .436$$

b. P(same numbers) = P(both selected balls are the same color)

= P(both red) + P(both green) = $.436 + \dfrac{4}{11} \bullet \dfrac{4}{10} = .581$

52. Let A_1 be the event that #1 fails and A_2 be the event that #2 fails. We assume that $P(A_1) = P(A_2) = q$ and that $P(A_1 \mid A_2) = P(A_2 \mid A_1) = r$. Then one approach is as follows:

$P(A_1 \cap A_2) = P(A_2 \mid A_1) \bullet P(A_1) = rq = .01$

$P(A_1 \cup A_2) = P(A_1 \cap A_2) + P(A_1' \cap A_2) + P(A_1 \cap A_2') = rq + 2(1-r)q = .07$

These two equations give 2q - .01 = .07, from which q = .04 and r = .25. Alternatively, with t = $P(A_1' \cap A_2) = P(A_1 \cap A_2')$, t + .01 + t = .07, implying t = .03 and thus q = .04 without reference to conditional probability.

53. P(B | A) = $\dfrac{P(A \cap B)}{P(A)} = \dfrac{P(B)}{P(A)}$ (since B is contained in A, A ∩ B = B)

$$= \frac{.05}{.60} = .0833$$

54. $P(A_1) = .22$, $P(A_2) = .25$, $P(A_3) = .28$, $P(A_1 \cap A_2) = .11$, $P(A_1 \cap A_3) = .05$, $P(A_2 \cap A_3) = .07$, $P(A_1 \cap A_2 \cap A_3) = .01$

a. $P(A_2 \mid A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} = \dfrac{.11}{.22} = .50$

b. $P(A_2 \cap A_3 \mid A_1) = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \dfrac{.01}{.22} = .0455$

c. $P(A_2 \cup A_3 \mid A_1) = \dfrac{P[A_1 \cap (A_2 \cup A_3)]}{P(A_1)} = \dfrac{P[(A_1 \cap A_2) \cup (A_1 \cap A_3)]}{P(A_1)}$

$$= \frac{P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{.15}{.22} = .682$$

d. $P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3) = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} = \dfrac{.01}{.53} = .0189$

This is the probability of being awarded all three projects given that at least one project was awarded.

55.

a. $P(A\ \ B) = P(B|A) \bullet P(A) = \dfrac{2\times 1}{4\times 3} \times \dfrac{2\times 1}{6\times 5} = .0111$

b. P(two other H's next to their wives | J and M together in the middle)

$$\frac{P[(H-W.or.W-H)and(J-M.or.M-J)and(H-W.or.W-H)]}{P(J-M.or.M-J.in.the.middle)}$$

numerator = $\dfrac{4\times 1\times 2\times 1\times 2\times 1}{6\times 5\times 4\times 3\times 2\times 1} = \dfrac{16}{6!}$

denominator = $\dfrac{4\times 3\times 2\times 1\times 2\times 1}{6\times 5\times 4\times 3\times 2\times 1} = \dfrac{48}{6!}$

so the desired probability = $\dfrac{16}{48} = \dfrac{1}{3}$.

c. P(all H's next to W's | J & M together)
= P(all H's next to W's – including J&M)/P(J&M together)

$$=\frac{\frac{6\times1\times4\times1\times2\times1}{6!}}{\frac{5\times2\times1\times4\times3\times2\times1}{6!}}=\frac{48}{240}=.2$$

56. If P(B|A) > P(B), then P(B'|A) < P(B').
Proof by contradiction.
Assume P(B'|A) ≥ P(B').
Then 1 – P(B|A) ≥ 1 – P(B).
- P(B|A) ≥ – P(B).
P(B|A) ≤ P(B).
This contradicts the initial condition, therefore P(B'|A) < P(B').

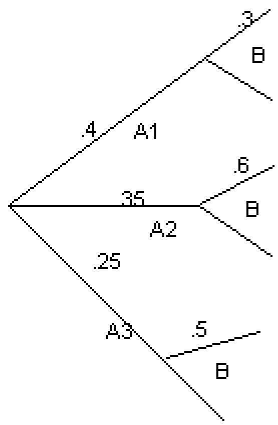
57. $$P(A\mid B)+P(A'\mid B)=\frac{P(A\cap B)}{P(B)}+\frac{P(A'\cap B)}{P(B)}=\frac{P(A\cap B)+P(A'\cap B)}{P(B)}=\frac{P(B)}{P(B)}=1$$

58. $$P(A\cup B\mid C)=\frac{P[(A\cup B)\cap C)}{P(C)}=\frac{P[(A\cap C)\cup(B\cap C)]}{P(C)}$$

$$=\frac{P(A\cap C)+P(B\cap C)-P(A\cap B\cap C)}{P(C)}$$

= P(A|C) + P(B|C) – P(A ∩ B | C)

59.



$.4 \times .3 = .12 = P(A_1 \cap B) = P(A_1) \bullet P(B \mid A)$

$.35 \times .6 = .21 = P(A_2 \cap B)$

$.25 \times .5 = .125 = P(A_3 \cap B)$

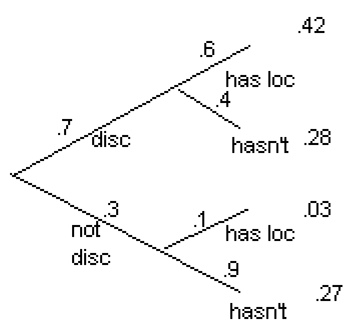
a. $P(A_2 \cap B) = .21$

b. $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = .455$

c. $P(A_1|B) = \dfrac{P(A_1 \cap B)}{P(B)} = \dfrac{.12}{.455} = .264$

$P(A_2|B) = \dfrac{.21}{.455} = .462$, $P(A_3|B) = 1 - .264 - .462 = .274$

60.



a. P(not disc | has loc) = $\dfrac{P(not.disc \cap has.loc)}{P(has.loc)} = \dfrac{.03}{.03 + .42} = .067$

b. P(disc | no loc) = $\dfrac{P(disc \cap no.loc)}{P(no.loc)} = \dfrac{.28}{.55} = .509$

61. P(0 def in sample | 0 def in batch) = 1

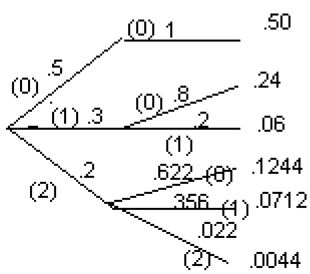
P(0 def in sample | 1 def in batch) = $\dfrac{\binom{9}{2}}{\binom{10}{2}} = .800$

P(1 def in sample | 1 def in batch) = $\dfrac{\binom{9}{1}}{\binom{10}{2}} = .200$

P(0 def in sample | 2 def in batch) = $\dfrac{\binom{8}{2}}{\binom{10}{2}} = .622$

P(1 def in sample | 2 def in batch) = $\dfrac{\binom{2}{1}\binom{8}{1}}{\binom{10}{2}} = .356$

P(2 def in sample | 2 def in batch) = $\dfrac{1}{\binom{10}{2}} = .022$



a. P(0 def in batch | 0 def in sample) = $\dfrac{.5}{.5+.24+.1244} = .578$

P(1 def in batch | 0 def in sample) = $\dfrac{.24}{.5+.24+.1244} = .278$

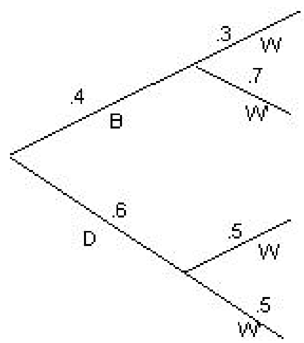
P(2 def in batch | 0 def in sample) = $\dfrac{.1244}{.5+.24+.1244} = .144$

b. P(0 def in batch | 1 def in sample) = 0

P(1 def in batch | 1 def in sample) = $\dfrac{.06}{.06+.0712} = .457$

P(2 def in batch | 1 def in sample) = $\dfrac{.0712}{.06+.0712} = .543$

62. Using a tree diagram, B = basic, D = deluxe, W = warranty purchase, W' = no warranty



$.4\times.3 = .12 = P(B\cap W)$

$.4\times.7 = .28 = P(B\cap W')$

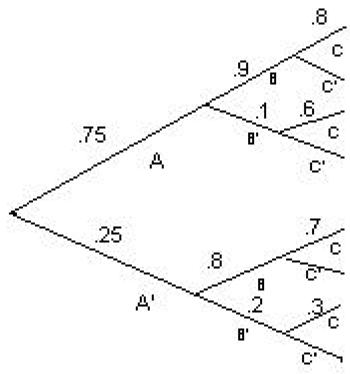
$.6\times.5 = .30 = P(D\cap W)$

$.6\times.5 = .30 = P(D\cap W')$

We want P(B|W) = $\dfrac{P(B\cap W)}{P(W)} = \dfrac{.12}{.30+.12} = \dfrac{.12}{.42} = .2857$

63.

a.



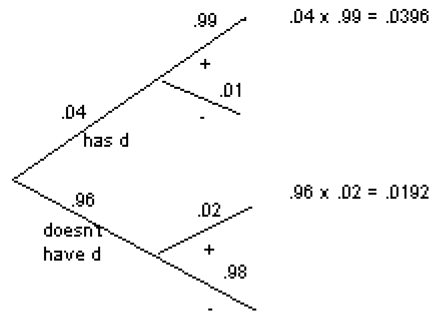
b. P(A $\cap$ B $\cap$ C) = .75 $\times$ .9 $\times$ .8 = .5400

c. P(B $\cap$ C) = P(A $\cap$ B $\cap$ C) + P(A′ $\cap$ B $\cap$ C)
=.5400+.25$\times$.8$\times$.7 = .6800

d. P(C) = P(A $\cap$ B $\cap$ C)+P(A′ $\cap$ B $\cap$ C) + P(A $\cap$ B′ $\cap$ C) + P(A′ $\cap$ B′ $\cap$ C)
= .54+.045+.14+.015 = .74

e. P(A|B $\cap$ C) = $\dfrac{P(A\cap B\cap C)}{P(B\cap C)} = \dfrac{.54}{.68} = .7941$

64.

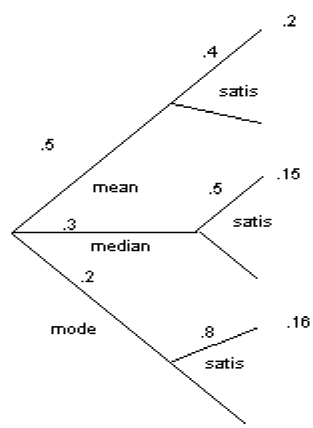


a. P(+) = .0588

b. P(has d | +) = $\dfrac{.0396}{.0588} = .6735$

c. P(doesn't have d | -) = $\dfrac{.9408}{.9412} = .9996$

65.



P(satis) = .51

P(mean | satis) = $\dfrac{.2}{.51} = .3922$

P(median | satis) = .2941

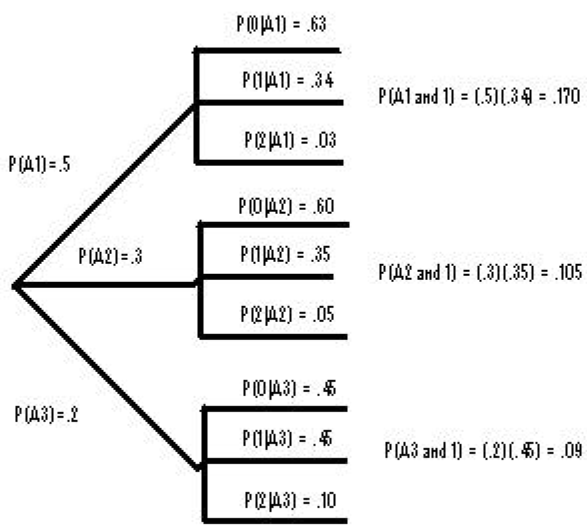
P(mode | satis) = .3137

So Mean (and not Mode!) is the most likely author, while Median is least.

66. Define events A1, A2, and A3 as flying with airline 1, 2, and 3, respectively. Events 0, 1, and 2 are 0, 1, and 2 flights are late, respectively. Event DC = the event that the flight to DC is late, and event LA = the event that the flight to LA is late. Creating a tree diagram as described in the hint, the probabilities of the second generation branches are calculated as follows: For the A1 branch, $P(0|A1) = P[DC' \cap LA'] = P[DC'] \cdot P[LA'] = (.7)(.9) = .63$; $P(1|A1) = P[(DC' \cap LA) \cup (DC \cap LA')] = (.7)(.1) + (.3)(.9) = .07 + .27 = .34$; $P(2|A1) = P[DC \cap LA] = P[DC] \cdot P[LA] = (.3)(.1) = .03$
Follow a similar pattern for A2 and A3.

From the law of total probability, we know that
$P(1) = P(A1 \cap 1) + P(A2 \cap 1) + P(A2 \cap 1)$
= (from tree diagram below) .170 + .105 + .09 = .365.

We wish to find P(A1|1), P(A2|1), and P(A2|1).

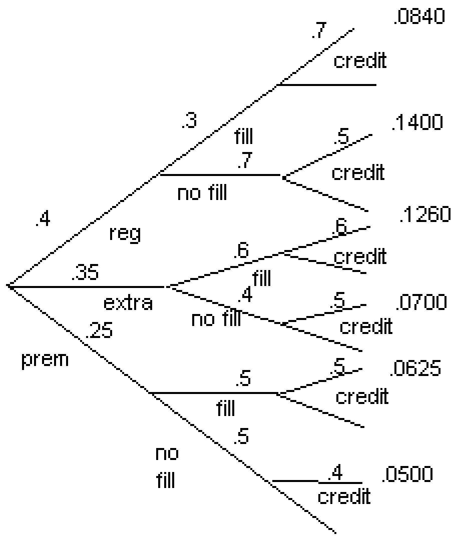


P(A1/1) = $\dfrac{P(A1 \cap 1)}{P(1)} = \dfrac{.170}{.365} = .466$;

P(A2/1) = $\dfrac{P(A2 \cap 1)}{P(1)} = \dfrac{.105}{.365} = .288$;

P(A3/1) = $\dfrac{P(A3 \cap 1)}{P(1)} = \dfrac{.090}{.365} = .247$;

67.



a. P(U ∩ F ∩ Cr) = .1260

b. P(Pr ∩ NF ∩ Cr) = .05

c. P(Pr ∩ Cr) = .0625 + .05 = .1125

d. P(F ∩ Cr) = .0840 + .1260 + .0625 = .2725

e. P(Cr) = .5325

f. P(PR | Cr) = $\dfrac{P(\text{Pr} \cap Cr)}{P(Cr)} = \dfrac{.1125}{.5325} = .2113$

Section 2.5

68. Using the definition, two events A and B are independent if $P(A|B) = P(A)$;
$P(A|B) = .6125$; $P(A) = .50$; $.6125 \neq .50$, so A and B are dependent.
Using the multiplication rule, the events are independent if
$P(A \cap B)=P(A) \bullet P(B)$;
$P(A \cap B) = .25$; $P(A) \bullet P(B) = (.5)(.4) = .2$. $.25 \neq .2$, so A and B are dependent.

69.

a. Since the events are independent, then A′ and B′ are independent, too. (see paragraph below equation 2.7. $P(B'|A') = .$ $P(B') = 1 - .7 = .3$

b. $P(A \cup B)=P(A) + P(B) - P(A)\cdot P(B) = .4 + .7 + (.4)(.7) = .82$

c. $P(AB'|\, A\cup B) = \dfrac{P(AB' \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(AB')}{P(A \cup B)} = \dfrac{.12}{.82} = .146$

70. $P(A_1 \cap A_2) = .11$, $P(A_1) \bullet P(A_2) = .055$. A_1 and A_2 are not independent.
$P(A_1 \cap A_3) = .05$, $P(A_1) \bullet P(A_3) = .0616$. A_1 and A_3 are not independent.
$P(A_2 \cap A_3) = .07$, $P(A_1) \bullet P(A_3) = .07$. A_2 and A_3 are independent.

71. $P(A' \cap B) = P(B) - P(A \cap B) = P(B) - P(A) \bullet P(B) = [1 - P(A)] \bullet P(B) = P(A') \bullet P(B)$.
Alternatively, $P(A' \mid B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{P(B) - P(A \cap B)}{P(B)}$

$$= \frac{P(B) - P(A)\cdot P(B)}{P(B)} = 1 - P(A) = P(A').$$

72. Using subscripts to differentiate between the selected individuals,
$P(O_1 \cap O_2) = P(O_1) \bullet P(O_2) = (.44)(.44) = .1936$
$P(\text{two individuals match}) = P(A_1 \cap A_2) + P(B_1 \cap B_2) + P(AB_1 \cap AB_2) + P(O_1 \cap O_2)$
$= .42^2 + .10^2 + .04^2 + .44^2 = .3816$

73. Let event E be the event that an error was signaled incorrectly. We want P(at least one signaled incorrectly) $= P(E_1 \cup E_2 \cup \ldots \cup E_{10}) = 1 - P(E_1' \cap E_2' \cap \ldots \cap E_{10}')$. $P(E') = 1 - .05 = .95$. For 10 independent points, $P(E_1' \cap E_2' \cap \ldots \cap E_{10}') = P(E_1')P(E_2')\ldots P(E_{10}')$ so $= P(E_1 \cup E_2 \cup \ldots \cup E_{10}) = 1 - [.95]^{10} = .401$. Similarly, for 25 points, the desired probability is $= 1 - [P(E')]^{25} = 1 - (.95)^{25} = .723$

74. P(no error on any particular question) = .9, so P(no error on any of the 10 questions) =$(.9)^{10}$ = .3487. Then P(at least one error) = 1 – $(.9)^{10}$ = .6513. For **p** replacing .1, the two probabilities are $(1-\mathbf{p})^n$ and $1 - (1-\mathbf{p})^n$.

75. Let q denote the probability that a rivet is defective.

a. P(seam need rework) = .20 = 1 – P(seam doesn't need rework)
= 1 – P(no rivets are defective)
= 1 – P(1^{st} isn't def $\cap \ldots \cap$ 25^{th} isn't def)
= $1 - (1 - q)^{25}$, so $.80 = (1 - q)^{25}$, $1 - q = (.80)^{1/25}$, and thus q = 1 - .99111 = .00889.

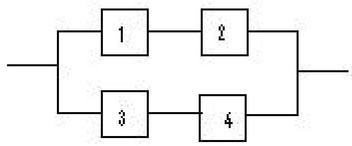
b. The desired condition is $.10 = 1 - (1 - q)^{25}$, i.e. $(1 - q)^{25} = .90$, from which q = 1 - .99579 = .00421.

76. P(at least one opens) = 1 – P(none open) = $1 - (.05)^5$ = .99999969
P(at least one fails to open) = 1 = P(all open) = $1 - (.95)^5$ = .2262

77. Let A_1 = older pump fails, A_2 = newer pump fails, and x = $P(A_1 \cap A_2)$. Then $P(A_1) = .10 + x$, $P(A_2) = .05 + x$, and $x = P(A_1 \cap A_2) = P(A_1) \bullet P(A_2) = (.10 + x)(.05 + x)$. The resulting quadratic equation, $x^2 - .85x + .005 = 0$, has roots x = .0059 and x = .8441. Hopefully the smaller root is the actual probability of system failure.

78. P(system works) = P(1 – 2 works $\cup$ 3 – 4 works)
= P(1 – 2 works) + P(3 – 4 works) - P(1 – 2 works $\cap$ 3 – 4 works)
= P(1 works $\cup$ 2 works) + P(3 works $\cap$ 4 works) – P(1 – 2) $\bullet$ P(3 – 4)
= (.9+.9-.81) + (.9)(.9) – (.9+.9-.81)(.9)(.9)
= .99 + .81 - .8019 = .9981

79.



Using the hints, let $P(A_i) = p$, and $x = p^2$, then P(system lifetime exceeds t_0) $= p^2 + p^2 - p^4 = 2p^2 - p^4 = 2x - x^2$. Now, set this equal to .99, or $2x - x^2 = .99 \Rightarrow x^2 - 2x + .99 = 0$. Use the quadratic formula to solve for x: $= \dfrac{2 \pm \sqrt{4-(4)(.99)}}{2} = \dfrac{2 \pm .2}{2} = 1 \pm .1 = .99 \text{ or } 1.01$

Since the value we want is a probability, and has to be = 1, we use the value of .99.

80. Event A: $\{ (3,1)(3,2)(3,3)(3,4)(3,5)(3,6) \}$, $P(A) = \frac{1}{6}$;

Event B: $\{ (1,4)(2,4)(3,4)(4,4)(5,4)(6,4) \}$, $P(B) = \frac{1}{6}$;

Event C: $\{ (1,6)(2,5)(3,4)(4,3)(5,2)(6,1) \}$, $P(C) = \frac{1}{6}$;

Event $A \cap B$: $\{ (3,4) \}$; $P(A \cap B) = \frac{1}{36}$;

Event $A \cap C$: $\{ (3,4) \}$; $P(A \cap C) = \frac{1}{36}$;

Event $B \cap C$: $\{ (3,4) \}$; $P(A \cap C) = \frac{1}{36}$;

Event $A \cap B \cap C$: $\{ (3,4) \}$; $P(A \cap B \cap C) = \frac{1}{36}$;

$P(A) \cdot P(B) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = P(A \cap B)$

$P(A) \cdot P(C) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = P(A \cap C)$

$P(B) \cdot P(C) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = P(B \cap C)$

The events are pairwise independent.

$P(A) \cdot P(B) \cdot P(C) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216} \neq \frac{1}{36} = P(A \cap B \cap C)$

The events are not mutually independent

81. P(both detect the defect) = 1 – P(at least one doesn't) = 1 - .2 = .8

a. P(1^{st} detects $\cap$ 2^{nd} doesn't) = P(1^{st} detects) – P(1^{st} does $\cap$ 2^{nd} does)
= .9 - .8 = .1
Similarly, P(1^{st} doesn't $\cap$ 2^{nd} does) = .1, so P(exactly one does)= .1+.1= .2

b. P(neither detects a defect) = 1 – [P(both do) + P(exactly 1 does)]
= 1 – [.8+.2] = 0
so P(all 3 escape) = (0)(0)(0) = 0.

82. P(pass) = .70

a. (.70)(.70)(.70) = .343

b. 1 – P(all pass) = 1 - .343 = .657

c. P(exactly one passes) = (.70)(.30)(.30) + (.30)(.70)(.30) + (.30)(.30)(.70) = .189

d. P(# pass $\le$ 1) = P(0 pass) + P(exactly one passes) = $(.3)^3$ + .189 = .216

e. P(3 pass | 1 or more pass) =

$$= \frac{P(3.pass \cap \ge 1.pass)}{P(\ge 1.pass)} = \frac{P(3.pass)}{P(\ge 1.pass)} = \frac{.343}{.973} = .353$$

83.

a. Let D_1 = detection on 1^{st} fixation, D_2 = detection on 2^{nd} fixation.
P(detection in at most 2 fixations) = $P(D_1) + P(D_1' \cap D_2)$
$= P(D_1) + P(D2 \mid D1')P(D_1)$
$= p + p(1 – p) = p(2 – p)$.

b. Define $D_1, D_2, \ldots, D_n$ as in **a**. Then P(at most n fixations)
$= P(D_1) + P(D_1' \cap D_2) + P(D_1' \cap D_2' \cap D_3) + \ldots + P(D_1' \cap D_2' \cap \ldots \cap D_{n-1}' \cap D_n)$
$= p + p(1 – p) + p(1 – p)^2 + \ldots + p(1 – p)^{n-1}$

$$= p\,[\,1 + (1 – p) + (1 – p)^2 + \ldots + (1 – p)^{n-1}] = p \bullet \frac{1-(1-p)^n}{1-(1-p)} = 1-(1-p)^n$$

Alternatively, P(at most n fixations) = 1 – P(at least n+1 are req'd)
= 1 – P(no detection in 1^{st} n fixations)
$= 1 – P(D_1' \cap D_2' \cap \ldots \cap D_n')$
$= 1 – (1 – p)^n$

c. P(no detection in 3 fixations) = $(1 – p)^3$

d. P(passes inspection) = P({not flawed} $\cup$ {flawed and passes})
= P(not flawed) + P(flawed and passes)
= .9 + P(passes | flawed)$\bullet$ P(flawed) = $.9+(1-p)^3(.1)$

e. P(flawed | passed) = $\dfrac{P(\textit{flawed} \cap \textit{passed})}{P(\textit{passed})} = \dfrac{.1(1-p)^3}{.9+.1(1-p)^3}$

For p = .5, P(flawed | passed) = $\dfrac{.1(.5)^3}{.9+.1(.5)^3} = .0137$

84.

a. P(A) = $\dfrac{2000}{10{,}000} = .02$, P(B) = P(A $\cap$ B) + P(A′ $\cap$ B)

= P(B|A) P(A) + P(B|A′) P(A′) = $\dfrac{1999}{9999} \bullet (.2) + \dfrac{2000}{9999} \bullet (.8) = .2$

P(A $\cap$ B) = .039984; since P(A $\cap$ B) $\neq$ P(A)P(B), the events are not independent.

b. P(A $\cap$ B) = .04. Very little difference. Yes.

c. P(A) = P(B) = .2, P(A)P(B) = .04, but P(A $\cap$ B) = P(B|A)P(A) = $\frac{1}{9} \cdot \frac{2}{10} = .0222$, so the two numbers are quite different.
In **a**, the sample size is small relative to the “population” size, while here it is not.

85. P(system works) = P(1 – 2 works $\cap$ 3 – 4 – 5 – 6 works $\cap$ 7 works)
= P(1 – 2 works) $\bullet$ P(3 – 4 – 5 – 6 works) $\bullet$P(7 works)
= (.99) (.9639) (.9) = .8588
With the subsystem in figure 2.14 connected in parallel to this subsystem,
P(system works) = .8588+.927 – (.8588)(.927) = .9897

86.

a. For route #1, P(late) = P(stopped at 2 or 3 or 4 crossings)

$= 1 - P(\text{stopped at 0 or 1}) = 1 - [.9^4 + 4(.9)^3(.1)]$

$= .0523$

For route #2, P(late) = P(stopped at 1 or 2 crossings)

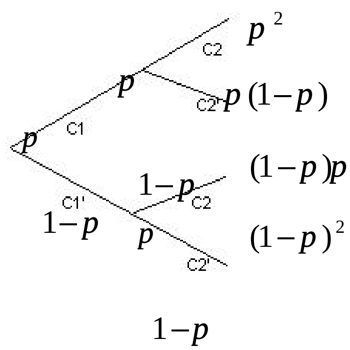
$= 1 - P(\text{stopped at none}) = 1 - .81 = .19$

thus route #1 should be taken.

b. P(4 crossing route | late) = $\dfrac{P(4crossing \cap late)}{P(late)}$

$$= \frac{(.5)(.0523)}{(.5)(.0523) + (.5)(.19)} = .216$$

87.



P(at most 1 is lost) = 1 – P(both lost)

$= 1 - \pi^2$

P(exactly 1 lost) = $2\pi(1 - \pi)$

P(exactly 1 | at most 1) = $\dfrac{P(exactly1)}{P(at.most1)} = \dfrac{2p(1-p)}{1-p^2}$

Supplementary Exercises

88.

a. $\binom{20}{3} = 1140$

b. $\binom{19}{3} = 969$

c. # having at least 1 of the 10 best = 1140 - # of crews having none of 10 best = 1140 - $\binom{10}{3} = 1140$ - 120 = 1020

d. P(best will not work) = $\frac{969}{1140} = .85$

89.

a. P(line 1) = $\frac{500}{1500} = .333$;

P(Crack) = $\frac{.50(500) + .44(400) + .40(600)}{1500} = \frac{666}{1500} = .444$

b. P(Blemish | line 1) = .15

c. P(Surface Defect) = $\frac{.10(500) + .08(400) + .15(600)}{1500} = \frac{172}{1500}$

P(line 1 and Surface Defect) = $\frac{.10(500)}{1500} = \frac{50}{1500}$

So P(line 1 | Surface Defect) = $= \frac{50/1500}{172/1500} = .291$

90.

a. The only way he will have one type of forms left is if they are all course substitution forms. He must choose all 6 of the withdrawal forms to pass to a subordinate. The desired probability is $\frac{\binom{6}{6}}{\binom{10}{6}}$ = .00476

b. He can start with the wd forms: W-C-W-C or with the cs forms: C-W-C-W:

# of ways: $6 \times 4 \times 5 \times 3 + 4 \times 6 \times 3 \times 5 = 2(360) = 720$;

The total # ways to arrange the four forms: $10 \times 9 \times 8 \times 7 = 5040$.

The desired probability is 720/5040 = .1429

91. P(A∪B) = P(A) + P(B) – P(A)P(B)
.626 = P(A) + P(B) - .144

So P(A) + P(B) = .770 and P(A)P(B) = .144.
Let x = P(A) and y = P(B), then using the first equation, y = .77 – x, and substituting this into the second equation, we get x (.77 – x) = .144 or
x^2 - .77x + .144 = 0. Use the quadratic formula to solve:

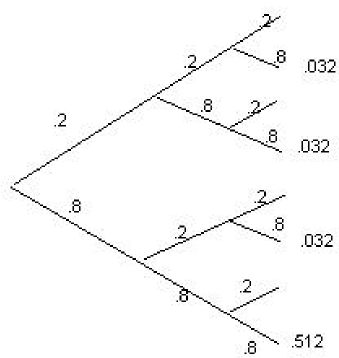
$$\frac{.77 \pm \sqrt{.77^2 - (4)(.144)}}{2} = \frac{.77 \pm .13}{2} = .32 \text{ or } .45$$

So P(A) = .45 and P(B) = .32

92.

a. (.8)(.8)(.8) = .512

b.



.512+.032+.023+.023 = .608

c. P(1 sent | 1 received) = $\dfrac{P(1sent \cap 1received)}{P(1received} = \dfrac{.4256}{.5432} = .7835$

93.

a. There are 5×4×3×2×1 = 120 possible orderings, so P(BCDEF) = $\frac{1}{120} = .0083$

b. # orderings in which F is 3rd = 4×3×1*×2×1 = 24, (* because F must be here), so P(F 3rd) = $\frac{24}{120} = .2$

c. P(F last) = $\dfrac{4\times 3\times 2\times 1\times 1}{120} = .2$

94. P(F hasn't heard after 10 times) = P(not on #1 ∩ not on #2 ∩…∩ not on #10)

$$= \left(\frac{4}{5}\right)^{10} = .1074$$

95. When three experiments are performed, there are 3 different ways in which detection can occur on exactly 2 of the experiments: (i) #1 and #2 and not #3 (ii) #1 and not #2 and #3; (iii) not#1 and #2 and #3. If the impurity is present, the probability of exactly 2 detections in three (independent) experiments is (.8)(.8)(.2) + (.8)(.2)(.8) + (.2)(.8)(.8) = .384. If the impurity is absent, the analogous probability is 3(.1)(.1)(.9) = .027. Thus

P(present | detected in exactly 2 out of 3) =

$$\frac{P(\det ected.in.exactly.2 \cap present)}{P(\det ected.in.exactly.2)}$$

$$= \frac{(.384)(.4)}{(.384)(.4) + (.027)(.6)} = .905$$

96. P(exactly 1 selects category #1 | all 3 are different)

$$= \frac{P(exactly.1.selects\#1 \cap all.are.different)}{P(all.are.different)}$$

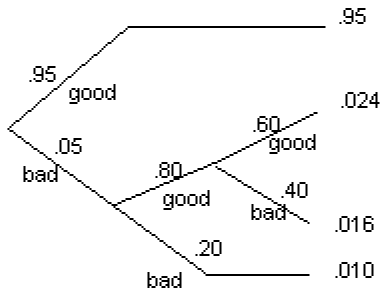
Denominator = $\dfrac{6\times 5\times 4}{6\times 6\times 6} = \dfrac{5}{9} = .5556$

Numerator = 3 P(contestant #1 selects category #1 and the other two select two different categories)

$$= 3\times\frac{1\times 5\times 4}{6\times 6\times 6} = \frac{5\times 4\times 3}{6\times 6\times 6}$$

The desired probability is then $\dfrac{5\times 4\times 3}{6\times 5\times 4} = \dfrac{1}{2} = .5$

97.



a. P(pass inspection) = P(pass initially $\cup$ passes after recrimping) = P(pass initially) + P(fails initially $\cap$ goes to recrimping $\cap$ is corrected after recrimping)
= .95 + (.05)(.80)(.60) (following path "bad-good-good" on tree diagram)
= .974

b. P(needed no recrimping | passed inspection) = $\dfrac{P(passed.initially)}{P(passed.inspection)}$

$$= \frac{.95}{.974} = .9754$$

98.

a. P(both +) = P(carrier $\cap$ both +) + P(not a carrier $\cap$ both +)
=P(both + | carrier) x P(carrier)
+ P(both + | not a carrier) x P(not a carrier)
= $(.90)^2(.01) + (.05)^2(.99) = .01058$
P(both –) = $(.10)^2(.01) + (.95)^2(.99) = .89358$
P(tests agree) = .01058 + .89358 = .90416

b. P(carrier | both + ve) = $\dfrac{P(carrier \cap both.positive)}{P(both.positive)} = \dfrac{(.90)^2(.01)}{.01058} = .7656$

99. Let A = 1st functions, B = 2nd functions, so P(B) = .9, P(A $\cup$ B) = .96, P(A $\cap$ B)=.75. Thus, P(A $\cup$ B) = P(A) + P(B) - P(A $\cap$ B) = P(A) + .9 - .75 = .96, implying P(A) = .81.

This gives P(B | A) = $\dfrac{P(B \cap A)}{P(A)} = \dfrac{.75}{.81} = .926$

100. $P(E_1 \cap \text{late}) = P(\text{late} \mid E_1)P(E_1) = (.02)(.40) = .008$

101.

a. The law of total probability gives

$$P(\text{late}) = \sum_{i=1}^{3} P(late \mid E_i) \cdot P(E_i)$$

$= (.02)(.40) + (.01)(.50) + (.05)(.10) = .018$

b. $P(E_1' \mid \text{on time}) = 1 - P(E_1 \mid \text{on time})$

$$= 1 - \frac{P(E_1 \cap on.time)}{P(on.time)} = 1 - \frac{(.98)(.4)}{.982} = .601$$

102. Let B denote the event that a component needs rework. Then

$$P(B) = \sum_{i=1}^{3} P(B \mid A_i) \cdot P(A_i) = (.05)(.50) + (.08)(.30) + (.10)(.20) = .069$$

Thus $P(A_1 \mid B) = \dfrac{(.05)(.50)}{.069} = .362$

$P(A_2 \mid B) = \dfrac{(.08)(.30)}{.069} = .348$

$P(A_3 \mid B) = \dfrac{(.10)(.20)}{.069} = .290$

103.

a. $P(\text{all different}) = \dfrac{(365)(364)...(356)}{(365)^{10}} = .883$

P(at least two the same) = 1 - .883 = .117

b. P(at least two the same) = .476 for k=22, and = .507 for k=23

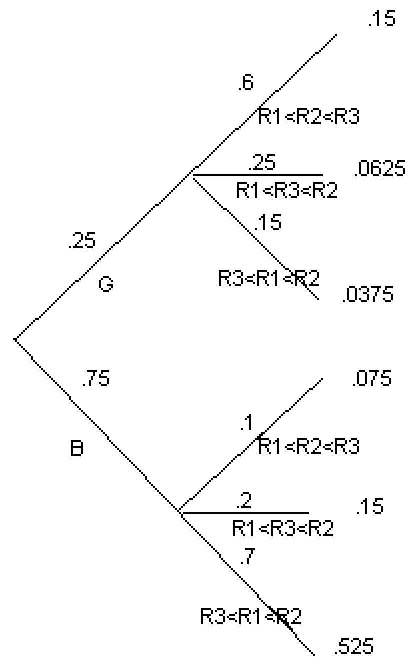
c. P(at least two have the same SS number) = 1 – P(all different)

$$= 1 - \frac{(1000)(999)...(991)}{(1000)^{10}}$$

$= 1 - .956 = .044$

Thus P(at least one "coincidence") = P(BD coincidence $\cup$ SS coincidence)
$= .117 + .044 - (.117)(.044) = .156$

104.



a. P(G | $R_1 < R_2 < R_3$) = $\dfrac{.15}{.15+.075} = .67$, P(B | $R_1 < R_2 < R_3$) = .33, classify as granite.

b. P(G | $R_1 < R_3 < R_2$) = $\dfrac{.0625}{.2125} = .2941$ < .05, so classify as basalt.

P(G | $R_3 < R_1 < R_2$) = $\dfrac{.0375}{.5625} = .0667$, so classify as basalt.

c. P(erroneous classif) = P(B classif as G) + P(G classif as B)
= P(classif as G | B)P(B) + P(classif as B | G)P(G)
= P($R_1 < R_2 < R_3$ | B)(.75) + P($R_1 < R_3 < R_2$ or $R_3 < R_1 < R_2$ | G)(.25)
= (.10)(.75) + (.25 + .15)(.25) = .175

d. For what values of p will $P(G \mid R_1<R_2<R_3) > .5$, $P(G \mid R_1 < R_3 < R_2) > .5$, $P(G \mid R_3 < R_1 < R_2) > .5$?

$P(G \mid R_1 < R_2 < R_3) = \dfrac{.6p}{.6p + .1(1-p)} = \dfrac{.6p}{.1 + .5p} > .5$ iff $p > \dfrac{1}{7}$

$P(G \mid R_1 < R_3 < R_2) = \dfrac{.25p}{.25p + .2(1-p)} > .5$ iff $p > \dfrac{4}{9}$

$P(G \mid R_3 < R_1 < R_2) = \dfrac{.15p}{.15p + .7(1-p)} > .5$ iff $p > \dfrac{14}{17}$ (most restrictive)

If $p > \dfrac{14}{17}$ always classify as granite.

105. P(detection by the end of the nth glimpse) = 1 – P(not detected in 1st n)

$= 1 - P(G_1' \cap G_2' \cap \ldots \cap G_n') = 1 - P(G_1')P(G_2') \ldots P(G_n')$

$= 1 - (1 - p_1)(1 - p_2) \ldots (1 - p_n) = 1 - \prod_{i=1}^{n}(1 - p_i)$

106.

a. P(walks on 4th pitch) = P(1st 4 pitches are balls) = $(.5)^4 = .0625$

b. P(walks on 6th) = P(2 of the 1st 5 are strikes, #6 is a ball)
= P(2 of the 1st 5 are strikes)P(#6 is a ball)
= $[10(.5)^5](.5) = .15625$

c. P(Batter walks) = P(walks on 4th) + P(walks on 5th) + P(walks on 6th)
= .0625 + .15625 + .15625 = .375

d. P(first batter scores while no one is out) = P(first 4 batters walk)
$=(.375)^4 = .0198$

107.

a. P(all in correct room) = $\dfrac{1}{4 \times 3 \times 2 \times 1} = \dfrac{1}{24} = .0417$

b. The 9 outcomes which yield incorrect assignments are: 2143, 2341, 2413, 3142, 3412, 3421, 4123, 4321, and 4312, so P(all incorrect) = $\dfrac{9}{24} = .375$

108.

a. P(all full) = $P(A \cap B \cap C) = (.6)(.5)(.4) = .12$
P(at least one isn't full) = 1 – P(all full) = 1 - .12 = .88

b. P(only NY is full) = $P(A \cap B' \cap C') = P(A)P(B')P(C') = .18$
Similarly, P(only Atlanta is full) = .12 and P(only LA is full) = .08
So P(exactly one full) = .18 + .12 + .08 = .38

109. Note: s = 0 means that the very first candidate interviewed is hired. Each entry below is the candidate hired for the given policy and outcome.

Outcome	s=0	s=1	s=2	s=3	**Outcome**	s=0	s=1	s=2	s=3
1234	1	4	4	4	3124	3	1	4	4
1243	1	3	3	3	3142	3	1	4	2
1324	1	4	4	4	3214	3	2	1	4
1342	1	2	2	2	3241	3	2	1	1
1423	1	3	3	3	3412	3	1	1	2
1432	1	2	2	2	3421	3	2	2	1
2134	2	1	4	4	4123	4	1	3	3
2143	2	1	3	3	4132	4	1	2	2
2314	2	1	1	4	4213	4	2	1	3
2341	2	1	1	1	4231	4	2	1	1
2413	2	1	1	3	4312	4	3	1	2
2431	2	1	1	1	4321	4	3	2	1

s	0	1	2	3
P(hire#1)	$\frac{6}{24}$	$\frac{11}{24}$	$\frac{10}{24}$	$\frac{6}{24}$

So s = 1 is best.

110. P(at least one occurs) = 1 – P(none occur)
$= 1 - (1-p_1)(1-p_2)(1-p_3)(1-p_4)$
$= p_1p_2(1-p_3)(1-p_4) + \ldots + (1-p_1)(1-p_2)p_3p_4$
$+ (1-p_1)p_2p_3p_4 + \ldots + p_1p_2p_3(1-p_4) + p_1p_2p_3p_4$

111. $P(A_1)$ = P(draw slip 1 or 4) = ½; $P(A_2)$ = P(draw slip 2 or 4) = ½;
$P(A_3)$ = P(draw slip 3 or 4) = ½; $P(A_1 \cap A_2)$ = P(draw slip 4) = ¼;
$P(A_2 \cap A_3)$ = P(draw slip 4) = ¼; $P(A_1 \cap A_3)$ = P(draw slip 4) = ¼
Hence $P(A_1 \cap A_2) = P(A_1)P(A_2) = ¼$, $P(A_2 \cap A_3) = P(A_2)P(A_3) = ¼$,
$P(A_1 \cap A_3) = P(A_1)P(A_3) = ¼$, thus there exists pairwise independence
$P(A_1 \cap A_2 \cap A_3)$ = P(draw slip 4) = ¼ $\neq$ 1/8 = $P(A_1)p(A_2)P(A_3)$, so the events are not mutually independent.