CHAPTER 10

Section 10.1

1.

a. H_o will be rejected if $f \geq F_{.05,4,15} = 3.06$ (since I – 1 = 4, and I (J – 1) = (5)(3) = 15).

The computed value of F is $f = \frac{MSTr}{MSE} = \frac{2673.3}{1094.2} = 2.44$. Since 2.44 is not ≥ 3.06, H_o is not rejected. The data does not indicate a difference in the mean tensile strengths of the different types of copper wires.

b. $F_{.05,4,15} = 3.06$ and $F_{.10,4,15} = 2.36$, and our computed value of 2.44 is between those values, it can be said that .05 < p-value < .10.

2.

Type of Box	$\bar{x}$	s
1	713.00	46.55
2	756.93	40.34
3	698.07	37.20
4	682.02	39.87

Grand mean = 712.51

$$MSTr = \frac{6}{4-1}\left[(713.00-712.51)^2 + (756.93-712.51)^2 + (698.07-712.51)^2 + (682.02-712.51)^2\right] = 6{,}223.0604$$

$$MSE = \frac{1}{4}\left[(46.55)^2 + (40.34)^2 + (37.20)^2 + (39.87)^2\right] = 1{,}691.9188$$

$$f = \frac{MSTr}{MSE} = \frac{6{,}223.0604}{1{,}691.9188} = 3.678$$

$$F_{.05,3,20} = 3.10$$

3.678 > 3.10, so reject H_o. There is a difference in compression strengths among the four box types.

3. With $m_i =$ true average lumen output for brand i bulbs, we wish to test $H_0 : m_1 = m_2 = m_3$ versus $H_a :$ at least two m_i's are unequal.

$MSTr = \hat{s}_B^2 = \frac{591.2}{2} = 295.60$, $MSE = \hat{s}_W^2 = \frac{4773.3}{21} = 227.30$, so

$f = \frac{MSTr}{MSE} = \frac{295.60}{227.30} = 1.30$ For finding the p-value, we need degrees of freedom I – 1 = 2 and I (J – 1) = 21. In the 2nd row and 21st column of Table A.9, we see that $1.30 < F_{.10,2,21} = 2.57$, so the p-value > .10. Since .10 is not < .05 , we cannot reject H_o. There are no differences in the average lumen outputs among the three brands of bulbs.

4. $x_{\bullet\bullet} = IJ\bar{x}_{\bullet\bullet} = 32(5.19) = 166.08$, so $SST = 911.91 - \frac{(166.08)^2}{32} = 49.95$.

$SSTr = 8\left[(4.39 - 5.19)^2 + ... + (6.36 - 5.19)^2\right] = 20.38$, so

$SSE = 49.95 - 20.38 = 29.57$. Then $f = \frac{20.38/3}{29.57/28} = 6.43$. Since $6.43 \geq F_{.05,2,28} = 2.95$, $H_0 : m_1 = m_2 = m_3 = m_4$ is rejected at level .05. There are differences between at least two average flight times for the four treatments.

5. $m_i =$ true mean modulus of elasticity for grade i (i = 1, 2, 3). We test $H_0 : m_1 = m_2 = m_3$ vs. $H_a :$ at least two m_i's are unequal. Reject H_o if $f \geq F_{.01,2,27} = 5.49$. The grand mean = 1.5367,

$MSTr = \frac{10}{2}\left[(1.63 - 1.5367)^2 + (1.56 - 1.5367)^2 + (1.42 - 1.5367)^2\right] = .1143$

$MSE = \frac{1}{3}\left[(.27)^2 + (.24)^2 + (.26)^2\right] = .0660$, $f = \frac{MSTr}{MSE} = \frac{.1143}{.0660} = 1.73$. Fail to reject H_o. The three grades do not appear to differ.

6.

Source	Df	SS	MS	F
Treatments	3	509.112	169.707	10.85
Error	36	563.134	15.643	
Total	39	1,072.256		

$F_{.01,3,36} \approx F_{.01,3,30} = 4.51$. The computed test statistic value of 10.85 exceeds 4.51, so reject H_o in favor of H_a: at least two of the four means differ.

7.

Source	Df	SS	MS	F
Treatments	3	75,081.72	25,027.24	1.70
Error	16	235,419.04	14,713.69	
Total	19	310,500.76		

The hypotheses are $H_0 : m_1 = m_2 = m_3 = m_4$ vs. H_a : at least two m_i's are unequal. $1.70 < F_{.10,3,16} = 2.46$, so p-value > .10, and we fail to reject H_o.

8. The summary quantities are $x_{1\bullet} = 2332.5$, $x_{2\bullet} = 2576.4$, $x_{3\bullet} = 2625.9$, $x_{4\bullet} = 2851.5$, $x_{5\bullet} = 3060.2$, $x_{\bullet\bullet} = 13{,}446.5$, so CF = 5,165,953.21, SST = 75,467.58, SSTr = 43,992.55, SSE = 31,475.03, $MSTr = \frac{43{,}992.55}{4} = 10{,}998.14$, $MSE = \frac{31{,}475.03}{30} = 1049.17$ and $f = \frac{10{,}998.14}{1049.17} = 10.48$. (These values should be displayed in an ANOVA table as requested.) Since $10.48 \geq F_{.01,4,30} = 4.02$, $H_0 : m_1 = m_2 = m_3 = m_4 = m_5$ is rejected. There are differences in the true average axial stiffness for the different plate lengths.

9. The summary quantities are $x_{1\bullet} = 34.3$, $x_{2\bullet} = 39.6$, $x_{3\bullet} = 33.0$, $x_{4\bullet} = 41.9$, $x_{\bullet\bullet} = 148.8$, $\Sigma\Sigma x_{ij}^2 = 946.68$, so $CF = \frac{(148.8)^2}{24} = 922.56$,
$SST = 946.68 - 922.56 = 24.12$,
$SSTr = \frac{(34.3)^2 + ... + (41.9)^2}{6} - 922.56 = 8.98$, $SSE = 24.12 - 8.98 = 15.14$.

Source	Df	SS	MS	F
Treatments	3	8.98	2.99	3.95
Error	20	15.14	.757	
Total	23	24.12		

Since $3.10 = F_{.05,3,20} < 3.95 < 4.94 = F_{.01,3,20}$, $.01 < p-value < .05$ and H_o is rejected at level .05.

10.

a. $E\left(\overline{X}_{\bullet\bullet}\right)=\dfrac{\Sigma E\left(\overline{X}_{i\bullet}\right)}{I}=\dfrac{\Sigma m_i}{I}=m$.

b. $E\left(\overline{X}_{i\bullet}^{2}\right)=Var\left(\overline{X}_{i\bullet}\right)+\left[E\left(\overline{X}_{i\bullet}\right)\right]^2=\dfrac{s^2}{J}+m_i^2$.

c. $E\left(\overline{X}_{\bullet\bullet}^{2}\right)=Var\left(\overline{X}_{\bullet\bullet}\right)+\left[E\left(\overline{X}_{\bullet\bullet}\right)\right]^2=\dfrac{s^2}{IJ}+m^2$.

d. $E(SSTr)=E\left[J\Sigma\overline{X}_{i\bullet}^{2}-IJ\overline{X}_{\bullet\bullet}^{2}\right]=J\sum\left(\dfrac{s^2}{J+m_i^2}\right)-IJ\left(\dfrac{s^2}{IJ+m^2}\right)$

$=Is^2+J\Sigma m_i^2-s^2-IJm^2=(I-1)s^2+J\Sigma(m_i-m)^2$, so

$E(MSTr)=\dfrac{E(SSTr)}{I-1}=E\left[J\Sigma\overline{X}_{i\bullet}^{2}-IJ\overline{X}_{\bullet\bullet}^{2}\right]=s^2+J\sum\dfrac{(m_i-m)^2}{I-1}$.

e. When H_o is true, $m_1=\ldots=m_i=m$, so $\Sigma(m_i-m)^2=0$ and $E(MSTr)=s^2$. When H_o is false, $\Sigma(m_i-m)^2>0$, so $E(MSTr)>s^2$ (on average, MSTr overestimates s^2).

Section 10.2

11. $Q_{.05,5,15}=4.37$, $w=4.37\sqrt{\dfrac{272.8}{4}}=36.09$.

3	1	4	2	5
437.5	462.0	469.3	512.8	532.1

The brands seem to divide into two groups: 1, 3, and 4; and 2 and 5; with no significant differences within each group but all between group differences are significant.

12.

3	1	4	2	5
437.5	462.0	469.3	512.8	532.1

Brands 2 and 5 do not differ significantly from one another, but both differ significantly from brands 1, 3, and 4. While brands 3 and 4 do differ significantly, there is not enough evident to indicate a significant difference between 1 and 3 or 1 and 4.

13.

3	1	4	2	5
427.5	462.0	469.3	502.8	532.1

Brand 1 does not differ significantly from 3 or 4, 2 does not differ significantly from 4 or 5, 3 does not differ significantly from1, 4 does not differ significantly from 1 or 2, 5 does not differ significantly from 2, but all other differences (e.g., 1 with 2 and 5, 2 with 3, etc.) do appear to be significant.

14. I = 4, J = 8, so $Q_{.05,4,28} \approx 3.87$, $w = 3.87\sqrt{\dfrac{1.06}{8}} = 1.41$.

1	2	3	4
4.39	4.52	5.49	6.36

Treatment 4 appears to differ significantly from both 1 and 2, but there are no other significant differences.

15. $Q_{.01,4,36} = 4.75$, $w = 4.75\sqrt{\dfrac{15.64}{10}} = 5.94$.

2	1	3	4
24.69	26.08	29.95	33.84

Treatment 4 appears to differ significantly from both 1 and 2, but there are no other significant differences.

16.

a. Since the largest standard deviation (s_4 = 44.51) is only slightly more than twice the smallest (s_3 = 20.83) it is plausible that the population variances are equal (see text p. 406).

b. The relevant hypotheses are $H_0 : m_1 = m_2 = m_3 = m_4 = m_5$ vs. $H_a :$ at least two m_i's differ. With the given f of 10.48 and associated p-value of 0.000, we can reject H_o and conclude that there is a difference in axial stiffness for the different plate lengths.

c.

4	6	8	10	12
333.21	368.06	375.13	407.36	437.17

There is no significant difference in the axial stiffness for lengths 4, 6, and 8, and for lengths 6, 8, and 10, yet 4 and 10 differ significantly. Length 12 differs from 4, 6, and 8, but does not differ from 10.

17. $q = \Sigma c_i m_i$ where $c_1 = c_2 = .5$ and $c_3 = -1$, so $\hat{q} = .5\bar{x}_{1\bullet} + .5\bar{x}_{2\bullet} - \bar{x}_{3\bullet} = -.396$ and $\Sigma c_i^2 = 1.50$. With $t_{.025,6} = 2.447$ and MSE = .03106, the CI is (from 10.5 on page 418)

$$-.396 \pm (2.447)\sqrt{\frac{(.03106)(1.50)}{3}} = -.396 \pm .305 = (-.701, -.091).$$

18.

a. Let $m_i =$ true average growth when hormone #i is applied. $H_0 : m_1 = ... = m_5$ will be rejected in favor of $H_a :$ at least two m_i's differ if $f \ge F_{.05,4,15} = 3.06$. With

$\frac{x_{\bullet\bullet}^2}{IJ} = \frac{(278)^2}{20} = 3864.20$ and $\Sigma\Sigma x_{ij}^2 = 4280$, SST = 415.80.

$\frac{\Sigma x_{i\bullet}^2}{J} = \frac{(51)^2 + (71)^2 + (70)^2 + (46)^2 + (40)^2}{4} = 4064.50$, so SSTr = 4064.50 – 3864.20 = 200.3, and SSE = 415.80 – 200.30 = 215.50. Thus

$MSTr = \frac{200.3}{4} = 50.075$, $MSE = \frac{215.5}{15} = 14.3667$, and

$f = \frac{50.075}{14.3667} = 3.49$. Because $3.49 \ge 3.06$, reject H_o. There appears to be a difference in the average growth with the application of the different growth hormones.

b. $Q_{.05,5,15} = 4.37$, $w = 4.37\sqrt{\frac{14.3667}{4}} = 8.28$. The sample means are, in increasing order, 10.00, 11.50, 12.75, 17.50, and 17.75. The most extreme difference is 17.75 – 10.00 = 7.75 which doesn't exceed 8.28, so no differences are judged significant. Tukey's method and the F test are at odds.

19. MSTr = 140, error d.f. = 12, so $f = \frac{140}{SSE/12} = \frac{1680}{SSE}$ and $F_{.05,2,12} = 3.89$.

$w = Q_{.05,3,12}\sqrt{\frac{MSE}{J}} = 3.77\sqrt{\frac{SSE}{60}} = .4867\sqrt{SSE}$. Thus we wish $\frac{1680}{SSE} > 3.89$ (significance of f) and $.4867\sqrt{SSE} > 10$ (= 20 – 10, the difference between the extreme $\bar{x}_{i\bullet}$'s - so no significant differences are identified). These become $431.88 > SSE$ and $SSE > 422.16$, so SSE = 425 will work.

20. Now MSTr = 125, so $f = \frac{1500}{SSE}$, $w = .4867\sqrt{SSE}$ as before, and the inequalities become $385.60 > SSE$ and $SSE > 422.16$. Clearly no value of SSE can satisfy both inequalities.

21.

a. Grand mean = 222.167, MSTr = 38,015.1333, MSE = 1,681.8333, and f = 22.6. The hypotheses are $H_0 : m_1 = ... = m_6$ vs. H_a : at least two m_i's differ . Reject H$_o$ if $f \geq F_{.01,5,78}$ (but since there is no table value for $n_2 = 78$, use $f \geq F_{.01,5,60} = 3.34$) With $22.6 \geq 3.34$, we reject H$_o$. The data indicates there is a dependence on injection regimen.

b. Assume $t_{.005,78} \approx 2.645$

i) Confidence interval for $m_1 - \frac{1}{5}(m_2 + m_3 + m_4 + m_5 + m_6)$:

$$\Sigma c_i \bar{x}_i \pm t_{a/2,I(J-1)}\sqrt{\frac{MSE(\Sigma c_i^2)}{J}}$$

$$= -67.4 \pm (2.645)\sqrt{\frac{1,681.8333(1.2)}{14}} = (-99.16, -35.64).$$

ii) Confidence interval for $\frac{1}{4}(m_2 + m_3 + m_4 + m_5) - m_6$:

$$= 61.75 \pm (2.645)\sqrt{\frac{1,681.8333(1.25)}{14}} = (29.34, 94.16)$$

Section 10.3

22. Summary quantities are $x_{1\bullet} = 291.4$, $x_{2\bullet} = 221.6$, $x_{3\bullet} = 203.4$, $x_{4\bullet} = 227.5$, $x_{\bullet\bullet} = 943.9$, $CF = 49{,}497.07$, $\Sigma\Sigma x_{ij}^2 = 50{,}078.07$, from which $SST = 581$,

$$SSTr = \frac{(291.4)^2}{5} + \frac{(221.6)^2}{4} + \frac{(203.4)^2}{4} + \frac{(227.5)^2}{5} - 49{,}497.07$$

$= 49{,}953.57 - 49{,}497.07 = 456.50$, and $SSE = 124.50$. Thus

$MSTr = \dfrac{456.50}{3} = 152.17$, $MSE = \dfrac{124.50}{18-4} = 8.89$, and f = 17.12. Because $17.12 \geq F_{.05,3,14} = 3.34$, $H_0 : m_1 = \ldots = m_4$ is rejected at level .05. There is a difference in yield of tomatoes for the four different levels of salinity.

23. $J_1 = 5$, $J_2 = 4$, $J_3 = 4$, $J_4 = 5$, $\bar{x}_{1\bullet} = 58.28$, $\bar{x}_{2\bullet} = 55.40$, $\bar{x}_{3\bullet} = 50.85$, $\bar{x}_{4\bullet} = 45.50$, MSE = 8.89. With $W_{ij} = Q_{.05,4,14} \cdot \sqrt{\dfrac{MSE}{2}\left(\dfrac{1}{J_i} + \dfrac{1}{J_j}\right)} = 4.11\sqrt{\dfrac{8.89}{2}\left(\dfrac{1}{J_i} + \dfrac{1}{J_j}\right)}$,

$\bar{x}_{1\bullet} - \bar{x}_{2\bullet} \pm W_{12} = (2.88) \pm (5.81)$; $\bar{x}_{1\bullet} - \bar{x}_{3\bullet} \pm W_{13} = (7.43) \pm (5.81)*$;
$\bar{x}_{1\bullet} - \bar{x}_{4\bullet} \pm W_{14} = (12.78) \pm (5.48)*$; $\bar{x}_{2\bullet} - \bar{x}_{3\bullet} \pm W_{23} = (4.55) \pm (6.13)$;
$\bar{x}_{2\bullet} - \bar{x}_{4\bullet} \pm W_{24} = (9.90) \pm (5.81)*$; $\bar{x}_{3\bullet} - \bar{x}_{4\bullet} \pm W_{34} = (5.35) \pm (5.81)$;

*Indicates an interval that doesn't include zero, corresponding to m's that are judged significantly different.

4 3 2 1

This underscoring pattern does not have a very straightforward interpretation.

24.

Source	Df	SS	MS	F
Groups	3-1=2	152.18	76.09	5.56
Error	74-3=71	970.96	13.68	
Total	74-1=73	1123.14		

Since $5.56 \geq F_{.01,2,71} \approx 4.94$, reject $H_0 : m_1 = m_2 = m_3$ at level .01.

25.

a. The distributions of the polyunsaturated fat percentages for each of the four regimens must be normal with equal variances.

b. We have all the $\overline{X}_{i.}'s$, and we need the grand mean:

$$\overline{X}_{..} = \frac{8(43.0)+13(42.4)+17(43.1)+14(43.5)}{52} = \frac{2236.9}{52} = 43.017$$

$$SSTr = \sum J_i(\overline{x}_{i.} - \overline{x}_{..})^2 = 8(43.0-43.017)^2 + 13(42.4-43.017)^2$$
$$+17(43.1-43.017)^2 + 13(43.5-43.017)^2 = 8.334$$

and $MSTr = \frac{8.334}{3} = 2.778$

$SSTr = \sum (J_i - 1)s^2 = 7(1.5)^2 + 12(1.3)^2 + 16(1.2)^2 + 13(1.2)^2 = 77.79$ and

$MSE = \frac{77.79}{48} = 1.621$. Then $f = \frac{MSTr}{MSE} = \frac{2.778}{1.621} = 1.714$ Since $1.714 < F_{.10,3,50} = 2.20$, we can say that the p-value is > .10. We do not reject the null hypothesis at significance level .10 (or any smaller), so we conclude that the data suggests no difference in the percentages for the different regimens.

26.

a.

i:	1	2	3	4	5	6	
J_I:	4	5	4	4	5	4	
$x_{i\bullet}$:	56.4	64.0	55.3	52.4	85.7	72.4	$x_{\bullet\bullet} = 386.2$
$\overline{x}_{i\bullet}$:	14.10	12.80	13.83	13.10	17.14	18.10	$\Sigma\Sigma x_j^2 = 5850.20$

Thus SST = 113.64, SSTr = 108.19, SSE = 5.45, MSTr = 21.64, MSE = .273, f = 79.3.

Since $79.3 \geq F_{.01,5,20} = 4.10$, $H_0 : \mu_1 = ... = \mu_6$ is rejected.

b. The modified Tukey intervals are as follows: (The first number is $\overline{x}_{i\bullet} - \overline{x}_{j\bullet}$ and the second is $W_{ij} = Q_{.01} \cdot \sqrt{\frac{MSE}{2}\left(\frac{1}{J_i} + \frac{1}{J_j}\right)}$.)

Pair	Interval	Pair	Interval	Pair	Interval
1,2	1.30 ± 1.37	2,3	-1.03 ± 1.37	3,5	-3.31 ± 1.37 *
1,3	$.27 \pm 1.44$	2,4	$-.30 \pm 1.37$	3,6	-4.27 ± 1.44 *
1,4	1.00 ± 1.44	2,5	-4.34 ± 1.29 *	4,5	-4.04 ± 1.37 *
1,5	-3.04 ± 1.37 *	2,6	-5.30 ± 1.37 *	4,6	-5.00 ± 1.44 *
1,6	-4.00 ± 1.44 *	3,4	$.37 \pm 1.44$	5,6	$-.96 \pm 1.37$

Asterisks identify pairs of means that are judged significantly different from one another.

c. The 99% t confidence interval is $\Sigma c_i \bar{x}_{i\bullet} \pm t_{.005,I(J-1)} \sqrt{\dfrac{MSE\left(\Sigma c_i^2\right)}{J_i}}$.

$\Sigma c_i \bar{x}_{i\bullet} = \frac{1}{4}\bar{x}_{1\bullet} + \frac{1}{4}\bar{x}_{2\bullet} + \frac{1}{4}\bar{x}_{3\bullet} + 14\bar{x}_{4\bullet} - 12\bar{x}_{5\bullet} - \frac{1}{2}\bar{x}_{6\bullet} = -4.16$, $\dfrac{\left(\Sigma c_i^2\right)}{J_i} = .1719$,

MSE = .273, $t_{.005,20} = 2.845$. The resulting interval is

$-4.16 \pm (2.845)\sqrt{(.273)(.1719)} = -4.16 \pm .62 = (-4.78, -3.54)$. The interval in the answer section is a Scheffe' interval, and is substantially wider than the t interval.

27.

a. Let $m_i =$ true average folacin content for specimens of brand I. The hypotheses to be tested are $H_0 : m_1 = m_2 = m_3 = m_4$ vs. H_a : at least two m_i's differ.

$\Sigma\Sigma x_{ij}^2 = 1246.88$ and $\dfrac{x_{\bullet\bullet}^2}{n} = \dfrac{(168.4)^2}{24} = 1181.61$, so SST = 65.27.

$\dfrac{\Sigma x_{i\bullet}^2}{J_i} = \dfrac{(57.9)^2}{7} + \dfrac{(37.5)^2}{5} + \dfrac{(38.1)^2}{6} + \dfrac{(34.9)^2}{6} = 1205.10$, so

$SSTr = 1205.10 - 1181.61 = 23.49$.

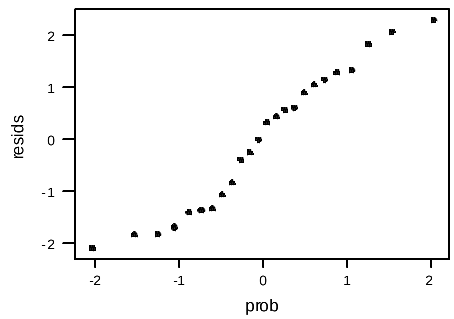
Source	Df	SS	MS	F
Treatments	3	23.49	7.83	3.75
Error	20	41.78	2.09	
Total	23	65.27		

With numerator df = 3 and denominator = 20,

$F_{.05,3,20} = 3.10 < 3.75 < F_{.01,3,20} = 4.94$, so $.01 < p-value < .05$, and since the p-value < .05, we reject H_o. At least one of the pairs of brands of green tea has different average folacin content.

b. With $\bar{x}_{i\bullet} = 8.27, 7.50, 6.35$, and 5.82 for I = 1, 2, 3, 4, we calculate the residuals $x_{ij} - \bar{x}_{i\bullet}$ for all observations. A normal probability plot appears below, and indicates that the distribution of residuals could be normal, so the normality assumption is plausible.

Normal Probability Plot for ANOVA Residuals



c. $Q_{.05,4,20} = 3.96$ and $W_{ij} = 3.96 \cdot \sqrt{\frac{2.09}{2}\left(\frac{1}{J_i} + \frac{1}{J_j}\right)}$, so the Modified Tukey intervals are:

Pair	Interval	Pair	Interval
1,2	$.77 \pm 2.37$	2,3	1.15 ± 2.45
1,3	1.92 ± 2.25	2,4	1.68 ± 2.45
1,4	2.45 ± 2.25 *	3,4	$.53 \pm 2.34$

4 3 2 1

Only Brands 1 and 4 are different from each other.

28. $$SSTr = \sum_i \left\{ \sum_j \left(\bar{X}_{i\bullet} - \bar{X}_{\bullet\bullet} \right)^2 \right\} = \sum_i J_i \left(\bar{X}_{i\bullet} - \bar{X}_{\bullet\bullet} \right)^2 = \sum_i J_i \bar{X}_{i\bullet}^2 - 2\bar{X}_{\bullet\bullet} \sum_i J_i \bar{X}_{i\bullet} + \bar{X}_{\bullet\bullet}^2 \sum_i J_i$$
$$= \sum_i J_i \bar{X}_{i\bullet}^2 - 2\bar{X}_{\bullet\bullet} X_{\bullet\bullet} + n\bar{X}_{\bullet\bullet}^2 = \sum_i J_i \bar{X}_{i\bullet}^2 - 2n\bar{X}_{\bullet\bullet}^2 + n\bar{X}_{\bullet\bullet}^2 = \sum_i J_i \bar{X}_{i\bullet}^2 - n\bar{X}_{\bullet\bullet}^2.$$

29. $E(SSTr) = E\left(\sum_i J_i \overline{X}_{i\bullet}^2 - n\overline{X}_{\bullet\bullet}^2\right) = \Sigma J_i E\left(\overline{X}_{i\bullet}^2\right) - nE\left(\overline{X}_{\bullet\bullet}^2\right)$

$= \Sigma J_i \left[Var\left(\overline{X}_{i\bullet}\right) + \left(E\left(\overline{X}_{i\bullet}\right)\right)^2\right] - n\left[Var\left(\overline{X}_{\bullet\bullet}\right) + \left(E\left(\overline{X}_{\bullet\bullet}\right)\right)^2\right]$

$= \Sigma J_i \left[\frac{s^2}{J_i} + m_i^2\right] - n\left[\frac{s^2}{n} + \frac{(\Sigma J_i m_i)^2}{n}\right]$

$= (I-1)s^2 + \Sigma J_i (m + a_i)^2 - [\Sigma J_i (m + a_i)]^2$

$= (I-1)s^2 + \Sigma J_i m^2 + 2m\Sigma J_i a_i + \Sigma J_i a_i^2 - [m\Sigma J_i]^2 \quad = (I-1)s^2 + \Sigma J_i a_i^2$, from which E(MSTr) is obtained through division by $(I-1)$.

30.

a. $a_1 = a_2 = 0$, $a_3 = -1$, $a_4 = 1$, so $\Phi^2 = \frac{2\left(0^2 + 0^2 + (-1)^2 + 1^2\right)}{1} = 4,\ \Phi = 2,$ and from figure (10.5), power $\approx .90$.

b. $\Phi^2 = .5J$, so $\Phi = .707\sqrt{J}$ and $n_2 = 4(J-1)$. By inspection of figure (10.5), J = 9 looks to be sufficient.

c. $m_1 = m_2 = m_3 = m_4, m_5 = m_1 + 1$, so $m = m_1 + \frac{1}{5}$, $a_1 = a_2 = a_3 = a_4 = -\frac{1}{5}$, $a_4 = \frac{4}{5}$, $\Phi^2 = \frac{2\left(20/25\right)}{1} = 1.60$ $\Phi = 1.26$, $n_1 = 4$, $n_2 = 45$. By inspection of figure (10.6), power $\approx .55$.

31. With $s = 1$ (any other s would yield the same Φ), $a_1 = -1,\ a_2 = a_3 = 0$, $a_4 = 1$, $\Phi^2 = \frac{.25\left(5(-1)^2 + 5(0)^2 + 5(0)^2 + 5(1)^2\right)}{1} = 2.5$, $\Phi = 1.58$, $n_1 = 3$, $n_2 = 14$, and power $\approx .62$.

32. With Poisson data, the ANOVA should be done using $y_{ij} = \sqrt{x_{ij}}$. This gives $y_{1\bullet} = 15.43$, $y_{2\bullet} = 17.15$, $y_{3\bullet} = 19.12$, $y_{4\bullet} = 20.01$, $y_{\bullet\bullet} = 71.71$, $\Sigma\Sigma y_{ij}^2 = 263.79$, CF = 257.12, SST = 6.67, SSTr = 2.52, SSE = 4.15, MSTr = .84, MSE = .26, f = 3.23. Since $F_{.01,3,16} = 5.29$, H_o cannot be rejected. The expected number of flaws per reel does not seem to depend upon the brand of tape.

33. $g(x) = x\left(1-\frac{x}{n}\right) = nu(1-u)$ where $u = \frac{x}{n}$, so $h(x) = \int [u(1-u)]^{-1/2}du$. From a table of integrals, this gives $h(x) = \arcsin\left(\sqrt{u}\right) = \arcsin\left(\sqrt{\frac{x}{n}}\right)$ as the appropriate transformation.

34. $E(MSTr) = s^2 + \frac{1}{I-1}\left(n - \frac{IJ^2}{n}\right)s_A^2 = s^2 + \frac{n-J}{I-1}s_A^2 = s^2 + Js_A^2$

Supplementary Exercises

35.

a. $H_0 : m_1 = m_2 = m_3 = m_4$ vs. H_a : at least two m_i's differ ; 3.68 is not $\geq F_{.01,3,20} = 4.94$, thus fail to reject H_o. The means do not appear to differ.

b. We reject H_o when the p-value < alpha. Since .029 is not < .01, we still fail to reject H_o.

36.

a. $H_0 : m_1 = ... = m_5$ will be rejected in favor of H_a : at least two m_i's differ if $f \geq F_{.05,4,40} = 2.61$. With $\bar{x}_{\bullet\bullet} = 30.82$, straightforward calculation yields $MSTr = \frac{221.112}{4} = 55.278$, $MSE = \frac{80.4591}{5} = 16.1098$, and $f = \frac{55.278}{16.1098} = 3.43$. Because $3.43 \geq 2.61$, H_o is rejected. There is a difference among the five teaching methods with respect to true mean exam score.

b. The format of this test is identical to that of part **a**. The calculated test statistic is $f = \frac{33.12}{20.109} = 1.65$. Since $1.65 < 2.61$, H_o is not rejected. The data suggests that with respect to true average retention scores, the five methods are not different from one another.

37. Let $m_i =$ true average amount of motor vibration for each of five bearing brands. Then the hypotheses are $H_0 : m_1 = ... = m_5$ vs. $H_a :$ at least two $m_i{'s}$ differ. The ANOVA table follows:

Source	Df	SS	MS	F
Treatments	4	30.855	7.714	8.44
Error	25	22.838	0.914	
Total	29	53.694		

$8.44 > F_{.001,4,25} = 6.49$, so p-value < .001, which is also < .05, so we reject H_o. At least two of the means differ from one another. The Tukey multiple comparison is appropriate. $Q_{.05,5,25} = 4.15$ (from Minitab output. Using Table A.10, approximate with $Q_{.05,5,24} = 4.17$). $W_{ij} = 4.15\sqrt{.914/6} = 1.620$.

Pair	$\bar{x}_{i\bullet} - \bar{x}_{j\bullet}$	Pair	$\bar{x}_{i\bullet} - \bar{x}_{j\bullet}$
1,2	-2.267*	2,4	1.217
1,3	0.016	2,5	2.867*
1,4	-1.050	3,4	-1.066
1,5	0.600	3,5	0.584
2,3	2.283*	4,5	1.650*

*Indicates significant pairs.

5 3 1 4 2

38. $x_{1\bullet} = 15.48$, $x_{2\bullet} = 15.78$, $x_{3\bullet} = 12.78$, $x_{4\bullet} = 14.46$, $x_{5\bullet} = 14.94$ $x_{\bullet\bullet} = 73.44$, so $CF = 179.78$, SST = 3.62, SSTr = 180.71 – 179.78 = .93, SSE = 3.62 - .93 = 2.69.

Source	Df	SS	MS	F
Treatments	4	.93	.233	2.16
Error	25	2.69	.108	
Total	29	3.62		

$F_{.05,4,25} = 2.76$. Since 2.16 is not ≥ 2.76, do not reject H_o at level .05.

39. $\hat{q} = 2.58 - \frac{2.63+2.13+2.41+2.49}{4} = .165$, $t_{.025,25} = 2.060$, MSE = .108, and $\Sigma c_i^2 = (1)^2 + (-.25)^2 + (-.25)^2 + (-.25)^2 + (-.25)^2 = 1.25$, so a 95% confidence interval for q is $.165 \pm 2.060\sqrt{\frac{(.108)(1.25)}{6}} = .165 \pm .309 = (-.144,.474)$. This interval does include zero, so 0 is a plausible value for q.

40. $m_1 = m_2 = m_3, m_4 = m_5 = m_1 - s$, so $m = m_1 - \frac{2}{5}s$, $a_1 = a_2 = a_3 = \frac{2}{5}s$, $a_4 = a_5 = -\frac{3}{5}s$. Then $\Phi^2 = \frac{J}{I}\sum \frac{a_i^2}{s^2}$

$$= \frac{6}{5}\left[\frac{3\left(\frac{2}{5}s\right)^2}{s^2} + \frac{2\left(-\frac{3}{5}s\right)^2}{s^2}\right] = 1.632$$

and $\Phi = 1.28$, $n_1 = 4$, $n_2 = 25$. By inspection of figure (10.6), power $\approx .48$, so $b \approx .52$.

41. This is a random effects situation. $H_0 : s_A^2 = 0$ states that variation in laboratories doesn't contribute to variation in percentage. H_o will be rejected in favor of H_a if $f \geq F_{.05,3,8} = 4.07$. SST = 86,078.9897 – 86,077.2224 = 1.7673, SSTr = 1.0559, and SSE = .7114. Thus $f = \frac{1.0559/3}{.7114/8} = 3.96$, which is not ≥ 4.07, so H_o cannot be rejected at level .05. Variation in laboratories does not appear to be present.

42.

a. $m_i =$ true average CFF for the three iris colors. Then the hypotheses are $H_0 : m_1 = m_2 = m_3$ vs. $H_a :$ at least two m_i's differ. SST = 13,659.67 – 13,598.36 = 61.31, $SSTR = \left(\frac{(204.7)^2}{8} + \frac{(134.6)^2}{5} + \frac{(169.0)^2}{6}\right) - 13{,}598.36 = 23.00$ The ANOVA table follows:

Source	Df	SS	MS	F
Treatments	2	23.00	11.50	4.803
Error	16	38.31	2.39	
Total	18	61.31		

Because $F_{.05,2,16} = 3.63 < 4.803 < F_{.01,2,16} = 6.23$, .01 < p-value < .05, so we reject H_o. There are differences in CFF based on iris color.

b. $Q_{.05,3,16} = 3.65$ and $W_{ij} = 3.65 \cdot \sqrt{\frac{2.39}{2}\left(\frac{1}{J_i} + \frac{1}{J_j}\right)}$, so the Modified Tukey intervals are:

Pair	$\left(\bar{x}_{i\bullet} - \bar{x}_{j\bullet}\right) \pm W_{ij}$
1,2	-1.33 ± 2.27
1,3	-2.58 ± 2.15 *
2,3	-1.25 ± 2.42

Brown	Green	Blue
25.59	26.92	28.17

The CFF is only significantly different for Brown and Blue iris color.

43. $\sqrt{(I-1)(MSE)\left(F_{.05,I-1,n-I}\right)} = \sqrt{(2)(2.39)(3.63)} = 4.166$. For $m_1 - m_2$, $c_1 = 1$, $c_2 = -1$, and $c_3 = 0$, so $\sqrt{\sum \frac{c_i^2}{J_i}} = \sqrt{\frac{1}{8} + \frac{1}{5}} = .570$. Similarly, for $m_1 - m_3$, $\sqrt{\sum \frac{c_i^2}{J_i}} = \sqrt{\frac{1}{8} + \frac{1}{6}} = .540$; for $m_2 - m_3$, $\sqrt{\sum \frac{c_i^2}{J_i}} = \sqrt{\frac{1}{5} + \frac{1}{6}} = .606$, and for $.5m_2 + .5m_2 - m_3$, $\sqrt{\sum \frac{c_i^2}{J_i}} = \sqrt{\frac{.5^2}{8} + \frac{.5^2}{5} + \frac{(-1)^2}{6}} = .498$.

Contrast	Estimate	Interval
$m_1 - m_2$	25.59 – 26.92 = -1.33	$(-1.33) \pm (.570)(4.166) = (-3.70, 1.04)$
$m_1 - m_3$	25.59 – 28.17 = -2.58	$(-2.58) \pm (.540)(4.166) = (-4.83, -.33)$
$m_2 - m_3$	26.92 – 28.17 = -1.25	$(-1.25) \pm (.606)(4.166) = (-3.77, 1.27)$
$.5m_2 + .5m_2 - m_3$	-1.92	$(-1.92) \pm (.498)(4.166) = (-3.99, 0.15)$

The contrast between m_1 and m_3 since the calculated interval is the only one that does not contain the value (0).

44.

Source	Df	SS	MS	F	$F_{.05}$
Treatments	3	24,937.63	8312.54	1117.8	4.07
Error	8	59.49	7.44		
Total	11	24,997.12			

Because $1117.8 \geq 4.07$, $H_0: m_1 = m_2 = m_3 = m_4$ is rejected. $Q_{.05,4,8} = 4.53$, so $w = 4.53\sqrt{\frac{7.44}{3}} = 7.13$. The four sample means are $\bar{x}_{4\bullet} = 29.92$, $\bar{x}_{1\bullet} = 33.96$, $\bar{x}_{3\bullet} = 115.84$, and $\bar{x}_{2\bullet} = 129.30$. Only $\bar{x}_{1\bullet} - \bar{x}_{4\bullet} < 7.13$, so all means are judged significantly different from one another except for m_4 and m_1 (corresponding to PCM and OCM).

45. $Y_{ij} - \bar{Y}_{\bullet\bullet} = c\left(X_{ij} - \bar{X}_{\bullet\bullet}\right)$ and $\bar{Y}_{i\bullet} - \bar{Y}_{\bullet\bullet} = c\left(\bar{X}_{i\bullet} - \bar{X}_{\bullet\bullet}\right)$, so each sum of squares involving Y will be the corresponding sum of squares involving X multiplied by c^2. Since F is a ratio of two sums of squares, c^2 appears in both the numerator and denominator so cancels, and F computed from Y_{ij}'s = F computed from X_{ij}'s.

46. The ordered residuals are –6.67, -5.67, -4, -2.67, -1, -1, 0, 0, 0, .33, .33, .33, 1, 1, 2.33, 4, 5.33, 6.33. The corresponding z percentiles are –1.91, -1.38, -1.09, -.86, -.67, -.51, -.36, -.21, -.07, .07, .21, .36, .51, .67, .86, 1.09, 1.38, and 1.91. The resulting plot of the respective pairs (the Normal Probability Plot) is reasonably straight, and thus there is no reason to doubt the normality assumption.